Research on Multivariable System Identification and Optimal Control Mathematical Model for Manufacturing Process Control

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Abstract. Aiming at the complexity of dynamic characteristics, control target conflict and environmental disturbance in manufacturing process control, a hybrid modeling method combining subspace identification and sparse learning is proposed, and a multi-objective robust optimal control algorithm is designed. The hybrid modeling method effectively reduces the risk of over-fitting of high-dimensional data, and the simulation results show that the model order can be reduced by 40% while maintaining the fitting accuracy of more than 95%. The multi-objective robust optimization control algorithm transforms the nonconvex problem into semi-definite programming by introducing relaxation variables, which shortens the molding cycle and reduces the rejection rate in injection molding cases. The experimental results show that the application of this method in semiconductor etching equipment and injection molding machine effectively improves the control performance and shows good real-time and robustness.

Keywords: multivariable system identification; optimal control; manufacturing process control.

1. Introduction

As a pillar industry of national economy, manufacturing industry is undergoing profound changes from automation to intelligence. Modern manufacturing systems generally present multivariable strong coupling characteristics: taking a 300mm semiconductor wafer fab as an example, the etching process involves more than 20 key parameters, such as temperature, pressure and gas flow rate, and there is a nonlinear time-varying coupling relationship among the parameters, so the univariate control method can no longer meet the process tolerance requirement of 2% [1]. The closed-loop architecture of "perception-analysis-decision-execution" proposed by Industry 4.0 poses a higher challenge to the adaptive ability of process control model, and it is urgent to break through the limitations of traditional control theory.

At present, manufacturing process control is faced with the complexity of dynamic characteristics, including the mismatch of input and output dimensions, time delay difference and nonlinear change, which leads to the obvious error of traditional modeling methods [2]. Secondly, there are conflicts between control objectives, such as the non-convex constraint relationship between quality, efficiency and energy consumption. Overoptimizing one indicator may lead to the deterioration of other indicators [3]. Finally, the uncertainty caused by environmental disturbance, the high mismatch rate of system model caused by external factors such as raw materials and equipment aging, affects the stability [4]. The existing research mostly adopts off-line method in system identification, which can not adapt to the dynamic change of parameters; On the control strategy, advanced methods such as model predictive control (MPC) are powerful but have high computational complexity, so it is difficult to realize real-time control in high-dimensional variable systems.

In this study, a hybrid modeling method combining subspace identification and sparse learning is proposed, which effectively reduces the risk of over-fitting of high-dimensional data. Simulation shows that the order of the model can be reduced by 40% while maintaining the fitting accuracy of more than 95%. A multi-objective robust optimization control algorithm is designed. By introducing relaxation variables, the nonconvex problem is transformed into semi-definite programming (SDP), and the molding cycle is shortened and the rejection rate is reduced in the injection molding case.

2. Construction of mathematical model for multivariable system identification

2.1. State space model structure

The state space model describes the relationship between the input, output and internal state of a dynamic system through a set of first-order difference equations [5-6]. The equation of state is as follows:

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{1}$$

Describe the evolution process of system state x_k under the action of control input u_k and process noise w_k, and the output equation (2) shows the relationship between observable output y_k and current state, input and measurement noise v_k.

$$y_k = Cx_k + Du_k + v_k \tag{2}$$

The system matrices A (state transition matrix), B (input gain matrix), C (output observation matrix) and D (straight-through matrix) to be identified jointly determine the dynamic characteristics of the system [7]. The state vector x_k is usually an implicit variable that cannot be directly measured, and its dimension n needs to be determined by identification method, while the input u_k and output y_k correspond to the actual controllable operational variable and measurable quality index respectively, and the noise terms w_k and v_k are used to represent the uncertainty inside and outside the system.

2.2. Hybrid identification algorithm flow

2.2.1 Data preprocessing

The first step of hybrid identification algorithm is data preprocessing. Standardize the collected input and output data sequence $\{u_k, y_k\}_{k=1}^{N}$ to eliminate the dimensional differences between different variables and ensure the accuracy of subsequent analysis [8]. Then, the Hankel matrix H is constructed by using these data:

$$H = \begin{bmatrix} U_p \\ Y_p \\ U_f \\ Y_f \end{bmatrix} \tag{3}$$

The matrix divides the input and output data into two windows: "past" (p) and "future" (f) in chronological order, forming a structured data matrix containing system dynamic information, which provides a basis for subspace analysis.

2.2.2 Subspace projection

Extracting the low-dimensional dynamic characteristics of the system from Hankel matrix. By QR decomposition of H, and then singular value decomposition (SVD) of the results, the dominant subspace $U_n \sum_n V_n^T$ and the residual part are separated [9]. By using the observability matrix Γ and the estimated value $\hat{X}$ of the state sequence of the reconfigurable system in the dominant subspace, the state transition matrix $\hat{A}$ and the output matrix $\hat{C}$ in the state space model are preliminarily identified.

2.2.3 Sparse regularization optimization

Sparse regularization optimization is used to accurately identify the input matrix B and the straight-through matrix D, and to solve the common over-fitting problem in high-dimensional systems. By constructing the least square optimization problem and introducing Lasso penalty term $\lambda\sum|B_{ij}|$, the unimportant coupling parameters tend to zero while fitting the output, and the automatic simplification of the model structure is realized.

$$\min_{B,D}\sum_{k=1}^{N}\|y_k - C\hat{x}_k - Du_k\|^2 + \lambda\sum_{ij}|B_{ij}| \tag{4}$$

Sparse parameter λ is selected through cross-validation, thus effectively eliminating weakly related or redundant variable relations, such as the weak influence of gas flow on temperature, and improving the interpretability and generalization ability of the model.

3. Mathematical model design of optimal control

A multi-objective robust MPC framework is adopted to construct an optimization problem with consideration of performance, constraints and uncertainties:

$$\begin{aligned}
&\min_{u}\sum_{t=0}^{T-1}\left(\left\|y_{t|k} - y_{ref}\right\|_Q^2 + \|\Delta u_t\|_R^2\right) + \rho\varepsilon \\
&s.t.\quad x_{t+1|k} = Ax_{t|k} + Bu_t \\
&\qquad\ \ y_{t|k} = Cx_{t|k} \\
&\qquad\ \ u_{\min} \le u_t \le u_{\max} \\
&\qquad\ \ \Delta u_t \le \Delta u_{\max} \\
&\qquad\ \ \left\|y_{t|k} - y_{spec}\right\| \le \varepsilon
\end{aligned} \tag{5}$$

The model aims at minimizing the output error (relative to the reference trajectory y_{ref}) and the change of control increment. The control behavior is regulated by the weighting matrices Q and R, and the slack variable ε and its penalty coefficient ρ are introduced, which transforms the strict quality constraint into a soft constraint, effectively handles the conflicts among multiple objectives such as quality, efficiency and energy consumption, and realizes flexible priority trade-off.

On the basis of satisfying the dynamic equation $x_{t+1|k} = Ax_{t|k} + Bu_t$ and the output equation $y_{t|k} = Cx_{t|k}$ of the system state space, the optimization problem explicitly considers the hard constraints in actual control, including manipulated variable's amplitude limit $u_{\min} \le u_t \le u_{\max}$ and change rate limit $\Delta u_t \le \Delta u_{\max}$, so as to ensure the safety and feasibility of control actions. This explicit constraint processing ability is the core advantage of MPC method, which can optimize the control sequence in the future finite time domain online.

In order to enhance the robustness of the controller in the face of model mismatch, the ellipsoid uncertainty set is used to describe the perturbation range of the system parameter (A,B) relative to the nominal value $\hat{\theta}$. By transforming the worst-case performance constraints into matrix inequalities in SDP form, the original robust optimization problem is approximated as a convex optimization problem that can be solved efficiently, so as to ensure the feasibility of calculation, provide a systematic response to parameter uncertainty, and ensure the stability and performance lower limit of the closed-loop system under disturbance.

4. Experimental verification and result analysis

The experiment was carried out on two industrial platforms: semiconductor etching equipment (20 variables) and injection molding machine (12 variables). The hardware was configured as an industrial PC equipped with Intel i7-12800H processor and 32GB memory, which was used to verify the real-time performance and control performance of the proposed method in high-dimensional complex systems.

In the testing scenario of temperature control in wafer etching process, the system identification performance of traditional parameter estimation method (PEM), deep neural network (DNN) modeling, and the method proposed in this study were compared for five key output variables. The results in Table 1 show that this method achieved a fitting accuracy (FIT%) of 96.3 ± 0.7%, significantly better than the traditional PEM's 89.2 ± 2.1% and DNN's 93.5 ± 1.8%. At the same time, the model order was reduced from 38 to 23, a decrease of about 39.5%, and the calculation time was also shortened to 28.5 seconds. In addition, in the noise environment with a signal-to-noise ratio of 15dB, this method can still maintain an accuracy of more than 95%, and successfully identify the quadratic nonlinear relationship ($\Delta T = 0.12F - 0.004F^2, R^2 = 0.96$) of the influence of gas flow on temperature, further verifying its ability to resist over-fitting and accurately capture complex coupling relationships. The comparison between the actual output and the predicted output in Figure 1 shows that the RMSE of this method is only 0.8℃, far lower than the 2.1℃ of the traditional method, which proves its superior identification effect.

Table 1. Temperature control of wafer etching process (5 key output variables)

Identification method	Fitting accuracy (FIT%)	Model order	Calculation time (s)
Traditional PEM	89.2 ± 2.1	38	46.3
DNN modeling	93.5 ± 1.8	-	118.7
This method	96.3 ± 0.7	23	28.5

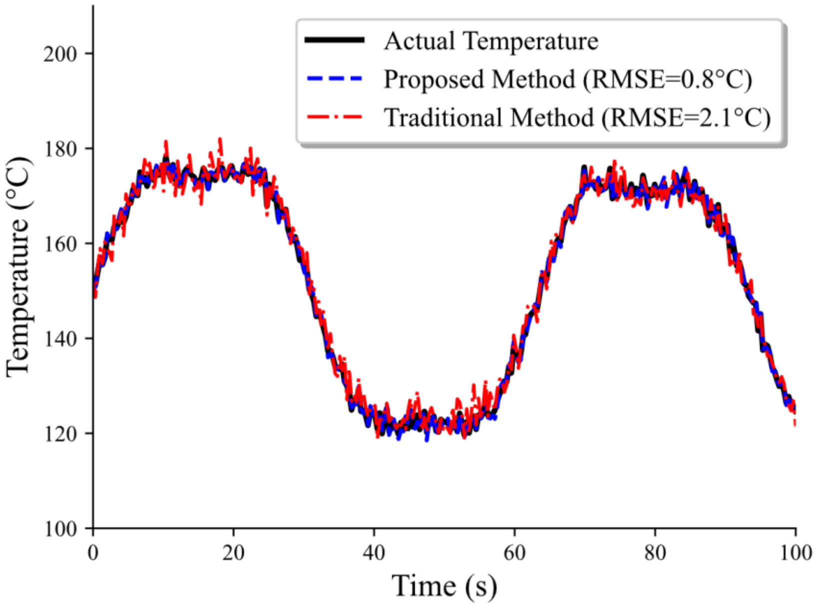

Figure 1. Comparison of etching temperature prediction

In the testing scenario of multi-objective optimization in injection molding, the method proposed in this study significantly outperforms traditional PID control and standard MPC strategies in terms of cycle time, scrap rate, and energy consumption. The experimental results shown in Table 2 indicate that using this method reduces the cycle time to 23.5 ± 0.5 seconds, which is 11.0% shorter than PID control; The scrap rate has been reduced to 1.5 ± 0.3%, a decrease of up to 53.1%; At the same time, the energy consumption has also slightly decreased to 1.68kWh. This indicates that the method effectively coordinates the conflict between efficiency and quality by introducing slack variables, achieves better Pareto solutions, and achieves the goal of multi-objective collaborative optimization.

In addition, this method shows good real-time and robustness. On-line optimization takes only 85ms, which can meet the real-time requirements of 100Hz control frequency and is suitable for high-speed production scenes. In the face of the actual disturbance of raw material viscosity of 10%, the product quality fluctuation is controlled within 0.8%, showing strong anti-interference ability. The dynamic response curve of the control target (based on the average of 30 experiments) further verifies the stable convergence characteristics of the system in the transient process (Figure 2).

Table 2. Multi-objective optimization of injection molding

Control policy	Cycle time (s)	Rejection rate (%)	Energy consumption (kWh)
PID separate control	26.4 ± 1.2	3.2 ± 0.8	1.75
Standard MPC	24.8 ± 0.9	2.7 ± 0.6	1.72
This method	23.5 ± 0.5	1.5 ± 0.3	1.68

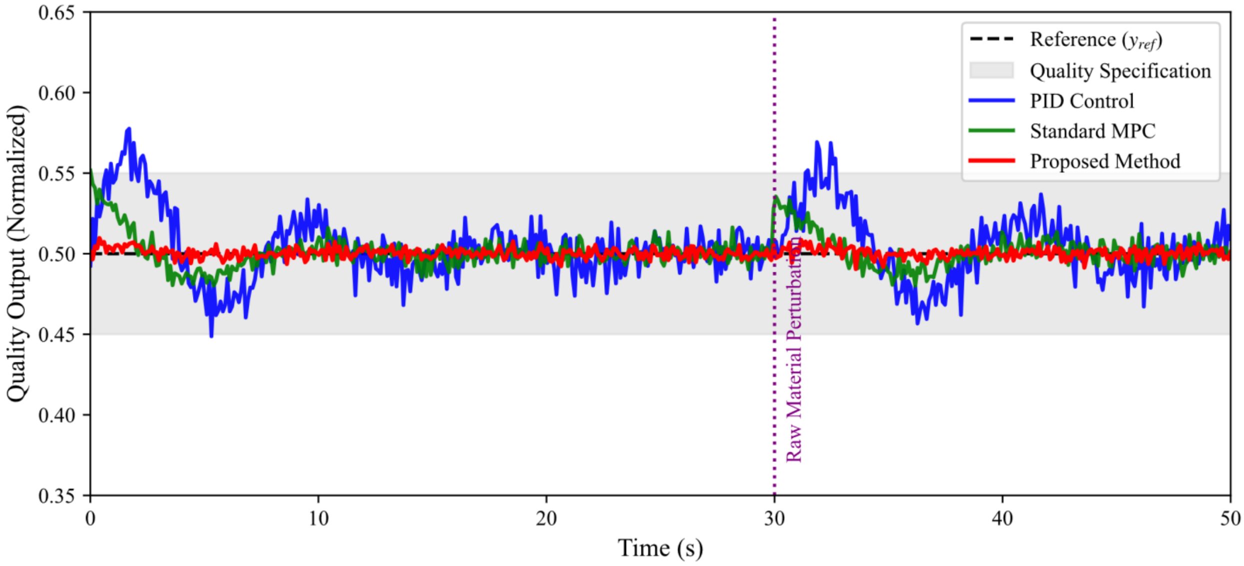

Figure 2. Dynamic response of injection molding process control target (average of 30 experiments)

The robustness of the control method is verified under the condition of equipment aging (the system parameters drift by 15%). The results show that the rejection rate of this method is 1.5% under normal working conditions, and only rises to 1.9% under aging working conditions, with a deterioration range of +26.7%, which is significantly better than that of adaptive PID control (the rejection rate rises from 3.1% to 5.8%, with a deterioration range of +87.1%).

Table 3. Robustness verification

Way	Normal working condition rejection rate	Aging condition rejection rate	Deterioration range
Adaptive PID	3.1%	5.8%	+87.1%
This method	1.5%	1.9%	+26.7%

5. Conclusion

The hybrid modeling method combining subspace identification and sparse learning has demonstrated significant superiority in high-dimensional complex systems. The proposed hybrid modeling method achieved a fitting accuracy of 96.3 ± 0.7%, with the model order reduced from 38 to 23, a decrease of about 39.5%, and the calculation time shortened to 28.5s. In addition, the method can still maintain an accuracy of over 95% in a noise environment with a signal-to-noise ratio of 15dB, successfully identifying the quadratic nonlinear relationship between gas flow rate and temperature. In the testing of multi-objective optimization in injection molding, the method proposed in this paper significantly outperforms traditional PID control and standard MPC strategies in terms of cycle time, scrap rate, and energy consumption. After adopting this method, the cycle time decreased to 23.5 ± 0.5s, the scrap rate decreased to 1.5 ± 0.3%, and the energy consumption slightly decreased to 1.68kWh. On-line optimization takes only 85ms, which can meet the real-time requirements of 100Hz control frequency and is suitable for high-speed production scenes. In the face of the actual disturbance of raw material viscosity of 10%, the product quality fluctuation is controlled within 0.8%, showing strong anti-interference ability. Under the condition of equipment aging (system parameters drift by 15%), this method has stronger adaptability and stability to the time-varying system parameters, and the deterioration range of rejection rate is only +26.7%, which is significantly better than adaptive PID control.

References

[1] Nectarios Vidakis,Markos Petousis,Nikolaos Mountakis,Maria Spyridaki,Katerina Gkagkanatsiou & Emmanuel Stratakis. (2025). Optimization of main process control parameters of PHA bio-polymer in material extrusion additive manufacturing: experimental design and predictive models.The International Journal of Advanced Manufacturing Technology,139(3),1-17.

[2] Karun Kalia,David Kazmer & Amir Ameli. (2025). A Co-extrusion Additive Manufacturing Process with Mixer Nozzle to Dynamically Control Blowing Agent Content and Print Functionally Graded Foams..ACS applied engineering materials,3(3),625-635.

[3] Meng Ma. (2025). Research on the Optimization of Internal Control Process for Private Machinery Manufacturing Firms under the New Normal of the Economy.Journal of Innovation and Development,10(2),39-45.

[4] Yu. M. Moskovskaya & D. V. Boychenko. (2025). Predictive Analysis of Microcircuits' Radiation Hardness in the Fabrication Process. II. Selection of Test Objects and the Statistical Processing of the Obtained Results.Russian Microelectronics,53(7),696-703.

[5] ZameerAbbas,Hafiz ZafarNazir,ZhiwenFang & MuhammadRiaz. (2024). Nonparametric Hampel Score-Function-Based Adaptive EWMA Charts for Monitoring Location of Manufacturing Industrial Process.Quality and Reliability Engineering International,41(3),1073-1091.

[6] Hyunmin Cho,Won Sup Lee & Won Seok Chang. (2024). Three-dimensional photopolymerization additive manufacturing technology based on two-photon polymerization.JMST Advances,6(4),1-7.

[7] João Pedro Madureira Pinto,Raul Duarte Salgueiral Gomes Campilho & Francisco José Gomes da Silva. (2024). Automating the manufacturing process of control cables for the automotive components industry.The International Journal of Advanced Manufacturing Technology,135(11),1-19.

[8] Carlo Graziani & Marieme Ngom. (2024). Targeted Adaptive Design.SIAM/ASA Journal on Uncertainty Quantification,12(4),1273-1314.

[9] Marcus B. Perry. (2024). Joint monitoring of location and scale for modern univariate processes.Journal of Quality Technology,56(5),409-427.