Analysis Of Staircase Wear Degree Based on Monte Carlo Simulation

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Abstract. Based on Archard's friction theory, an Archard model considering both anthropogenic wear (related to gait, material hardness, and wear coefficient) and environmental weathering wear is established to predict stair wear depth distribution. A probabilistic model for pedestrian footprint distribution is developed using the beta function (for footfall timing), two-dimensional normal distribution (for footfall center), and random step angle deflection. Monte Carlo simulation is applied to generate random pedestrian stepping events, simulating movement tendencies (up/down) and habits (single/side-by-side walking). Model validation shows good agreement between predicted wear volume (15540 cm^3) and actual data (15303 cm^3). Sensitivity analyses confirm model stability. Results indicate that step wear morphology can infer pedestrian habits, step age, and refurbishment history, providing valuable references for ancient site research.

Keywords: Archard Equation, Monte Carlo simulation, Wear depth, Normal distribution.

1. Introduction

Archaeology is an interdisciplinary field that uncovers historical cultures and social activities through the study of sites and cultural relics. To accurately obtain information such as the usage history, construction date, and cultural background of ancient buildings, archaeological methods are constantly integrated with emerging technologies, exhibiting diversified and interdisciplinary features [1].

Stairs, as common components in ancient buildings, act as a link connecting different floors. The pedestrian flow and movement patterns reflected by them in a certain past period have drawn the attention of archaeologists [2], such as issues concerning the direction of stair usage and the degree of concentration in their use.

Stairs in historical buildings are mostly made of stone and wood. After being trampled for a long time, they withstand various stresses and develop uneven wear marks. Archaeologists can gain information from the inversion of the surface morphology of these steps.

Freire-Lista D M and others analyzed the wear of granite stairs, studied the impact of human activities on changes in building surfaces, and introduced scanning and analysis technologies like surface scanners and 3D laser scanners to measure wear depth, rate, and regional distribution. They also provided a statistical physics model that uses quantitative descriptions of random loss to quantify the wear degree of stair surfaces [3]. The innovation of this paper lies in organically combining the Archard Equation with Monte Carlo simulation on the basis of existing research to construct a brand-new stair wear model.

Accurate, timely, and non-destructive sampling is of vital importance to archaeological research. When there are a large number of steps, the method of taking high-quality images for marking and evaluation is obviously time-consuming and labor-intensive, and experts may overestimate the surface properties of the stairs. Therefore, using scanning technology to obtain the surface morphology of stairs, generate point cloud data, and build a specific archaeological research cloud platform in combination with large models such as machine learning is one of the main mainstream directions in archaeological research [4]. Integrating the stair wear model that combines the Archard Equation and Monte Carlo simulation into this platform can leverage the platform's computing power to efficiently process and simulate a large amount of stair wear data, further improving the accuracy and reliability of wear prediction, and providing a more powerful quantitative basis for archaeologists to infer the usage history and pedestrian flow patterns of stairs.

2. Research on the stair wear model

2.1. Assumptions and Justifications

To simplify the problem and facilitate the simulation of real-lifeconditions, it is significant to make the following basic assumptions,each of which is justified.

First of all, all individuals using the stairs are standard healthy young adults (specifically, those without joint-related illnesses). Factors such as a person’s height, sex, and age influence their movement process when ascending or descending stairs. However, since this paper focuses on archaeological research from the perspective of step force analysis, the effects of these differences are not considered [4][5].

Second, the friction loss of steps is only related to the friction between the soles of passers-by’s shoes and the step surface during walking, as well as environmental weathering. Wear can be categorized into abrasive wear, adhesive wear, fatigue wear, and corrosive wear[6]. In real scenarios, multiple mechanisms coexist and interact. Given that this paper studies wear caused by pedestrian activity on steps, only the effects of adhesive wear and corrosive wear are considered.

Third, the human foot is approximately rectangular in shape with uniform size. Based on anatomical and biomechanical principles, the roughly rectangular shape and uniform size distribution of the human foot can be justified. From an overhead view, the width of the arch and the position of the toes give the human foot a generally rectangular appearance.

Finally, passers-by adopt a step-over-step walking strategy when using the stairs. People typically use three walking strategies—step-over-step, step-by-step, and side-step (as shown in Figure 1)—depending on factors such as age and weight [7]. Combined with Assumption 1 (focusing on healthy young adults) and common outdoor hiking habits, it is concluded that all studied passers-by use the step-over-step strategy.

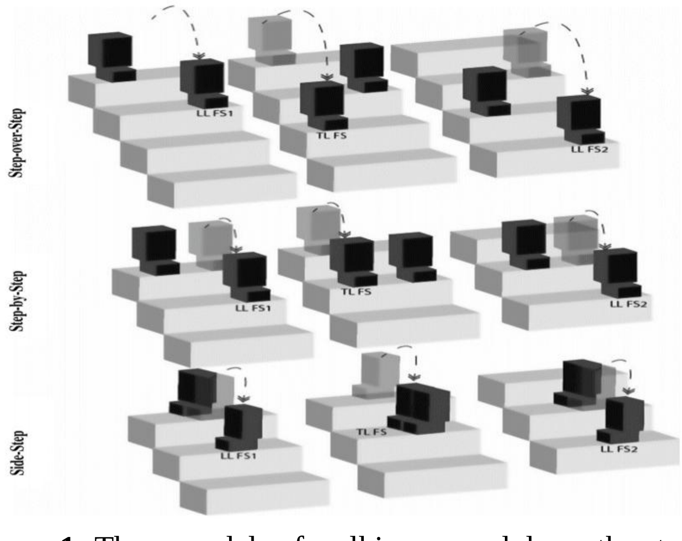

Figure 1: Three models of walking up and down the stairs

2.2. The Model for Predicting Changes in Stair Wear

In this section, the paper develops a modified Archard model that accounts for variations in ascending and descending stairs, wear rates over time, and the environment's natural weathering. This model is used to predict the deterioration of staircases in historic buildings over time.

Without considering the environment's natural weathering, the difference between climbing and descending stairs, or the change in wear rate over time, the optimal Archard model for wear variation is shown in Equation 1.

$$V_{\mathrm{h}} = k * \frac{F_n * L}{H} \tag{1}$$

Where V_{h} denotes the volume of wear due to human activity, k denotes the wear coefficient of the material, F_{n} denotes the normal load acting on the contact surface, L denotes the sliding distance, and H denotes the hardness of the material. When considering differences in ascending and descending staircases, a correction term η needs to be introduced, as shown in Equation 2.

$$V_{\mathrm{h}} = k * \frac{F_n * L}{H} * \eta \tag{2}$$

Where η represents the direction factor of going up and down the stairs.When $\eta > 1$,pedestrians tend to go up the stairs, causing more wear and tear on the stairs; when $\eta < 1$,pedestrians tend to go up the stairs, causing less wear and tear on the stairs.

The pace and frequency of stair climbing and descending, as well as the contact loads produced on the contact surfaces, also have an impact on the orientation factor. According to studies [5], pedestrians move more slowly and less frequently when ascending and descending stairs. Conversely, they move more quickly and frequently under certain conditions. Additionally, the loads that people place on the contact surface vary slightly depending on the direction of acceleration when moving up and down the steps. Equation 3 illustrates the relationship between climbing and descending stairs and the normal load F_{n} exerted by a person on the contact surface.

$$F_n - m * g = m * a (a = \pm 0.3) \tag{3}$$

Where a is the acceleration up and down the stairs, which is positive at the top and negative at the bottom. Since the gravitational acceleration g is much larger than a, and the difference between the step speed step frequency of going up and down the stairs is not large, therefore, a reasonable constant relationship is established between the direction factor and going up and down the stairs, as shown in Equation 4.

$$\eta = \begin{cases} 1.2 (up) \\ 0.8 (down) \end{cases} \tag{4}$$

The paper finds that the wear rate k changes over time. Since the historical buildings considered in this paper have a long-life cycle, the wear rate decreases the further back in time this paper goes. Therefore, the growth rate and precipitation are not simply constant. This paper assume that the wear rate k decreases exponentially with time t, as shown in Equation 5.

$$k(t) = k_0 * e^{-\beta * t} \tag{5}$$

In the formula, k represents the initial wear rate and β represents the gradient of influence between wear rate and time.

In addition, environmental weathering affects the degree of staircase wear, which is assumed to be a power function of time, as shown in Equation 6.

$$V_{\mathrm{e}} = \varepsilon_0 * (1 + \gamma * \mathrm{t}^{\mathrm{n}}) * S \tag{6}$$

Where γ is the gradient of the amount of wear caused by environmental weathering with respect to the effect of time, ε_0 is the ideal degree of environmental weathering under initial conditions, and S is the area of the step.

Substituting the expressions for the above parameters into Equation 2,This paper obtains equations describing the wear volume of the staircase as well as environmental factors, differences in ascending and descending the staircase, etc., and successfully construct a mathematical model. Finally, the total volume of wear versus time is shown in Equation 7.

$$V = V_{\mathrm{e}} + V_{\mathrm{h}} \tag{7}$$

2.3. Simulation of wear on a specific set of stairs

A popular modeling technique for discretizing continuous mathematical problems is lattice discretization. This paper suggests discretizing the stepped steps and counting the number of times each of these sections is stepped on because different pedestrians' stepping areas may overlap, making it challenging to solve for the overall volume of the worn area.

The paper divides the length and width of the steps into a grid of I rows and J columns according to the steps Δx and Δy. The schematic is shown in Figure 2, totaling I × J discrete regions, and for any one of these discrete regions A_{ij}, the center of which has the coordinates in the coordinate system of the steps:

$$\begin{pmatrix} x_{ij} \\ y_{ij} \end{pmatrix} = \begin{pmatrix} -\dfrac{e}{2} + i\Delta x \\ -\dfrac{r}{2} + j\Delta y \end{pmatrix} \tag{8}$$

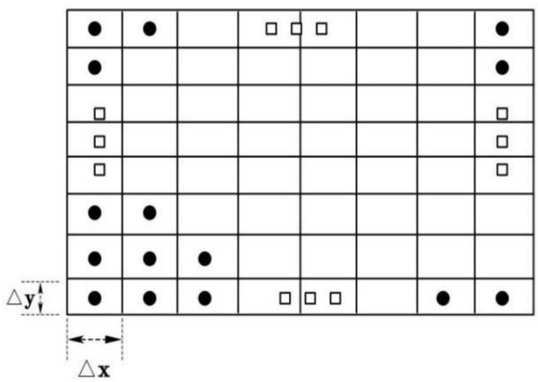

Figure 2: Schematic representation of the discretized step region

3. Determine walking posture

First, establish a Cartesian coordinate system for the staircase, with the center of the staircase as the origin, the horizontal plane as the X-axis, and the vertical plane as the Y-axis.

Then, when a person walks on a staircase, the vertical surfaces of the foot and the step are not in a parallel relationship, but there is a certain angle, called the gait angle. As shown in Figure 3, the angle between the foot and the vertical plane changes with each step. In the model of this paper, the foot is approximated as a rectangle and changes with the position of the foot center point, so in the foot surface coordinate system, the gait angle is shown in Figure 4[8].

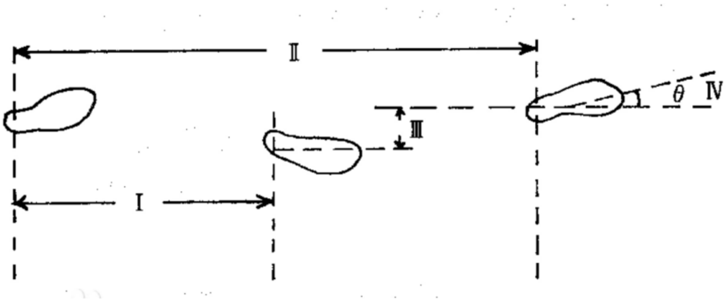

Figure 3: Graph of dynamic gait angle

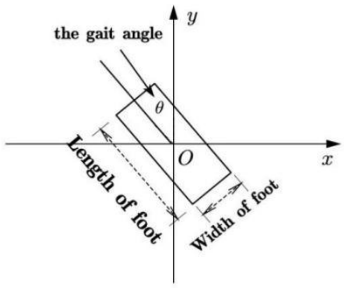

Figure 4: ait angle θ

3.1. Reasonable Estimation of Essential Parameters

Due to the footfall distribution model developed in Section 3.1.2, the location of the footfall distribution can be obtained from Equation 9. Due to $\zeta \sim (\mu, \sigma^2)$, it is important to determine the mean μ and variance σ^2.

Based on the probability property of normal distribution, this paper can deduce the simple but practical 3σ principle, that is most of the data will fall within the interval (μ - 3σ, μ + 3σ).

Thus,can better describe the concentration trend of the data and screen out outliers. To determine the normal distribution with preference differences in the y-direction for the two groups (upstairs and downstairs), This paper can utilize as many data points as possible by solving overdetermined equations, thereby improving the precision and stability of parameter estimation.

By collecting a large number of experimental results of pedestrians (taking walking down the stairs as an example), this paper can use the statistical law to determine the approximate range of y-coordinates of pedestrians' footprints left on the steps, which is obtained from experiments as [-11.733, -3.735]. These values correspond to the two limiting values at $\mu_y - 3\sigma_y$ and $\mu_y + 3\sigma_y$, thus deriving the Overdetermined Equation 13[9]:

$$\begin{cases} \mu_y - 3\sigma_y = -11.733 \\ \mu_y = -7.76 \\ \mu_y + 3\sigma_y = -3.735 \end{cases} \quad (9)$$

$$\left(\mu_y \quad \sigma_y\right)^T = (A^T A)^{-1} A^T B \quad (10)$$

Where A is the coefficient matrix of in Equation 14, B is $(-11.733, -7.76, -3.735)^T$.

Finally, it can be obtained that μ_γ = -7.74267 and σ_γ = 13.33. For the scenario of a single person walking, this paper employs the Monte Carlo method to simulate the position distribution of the pedestrian flow (following a normal distribution) tens of thousands of times. This simulation generates the rectangular area representing the human foot. Subsequently, using the Separating Axis Theorem, this paper determines the small squares that are stepped on by the foot each time. Finally, by counting the number of times each small square is stepped on, this paper obtains the schematic diagram as shown in Figure 5.

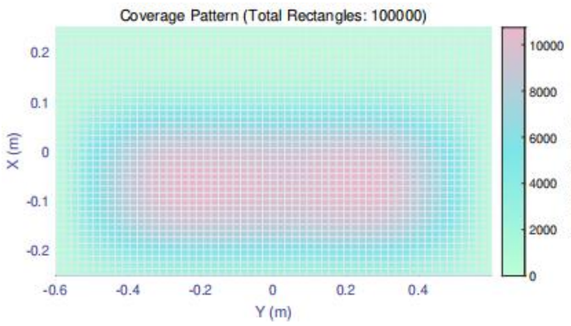

Figure 5: Walking alone

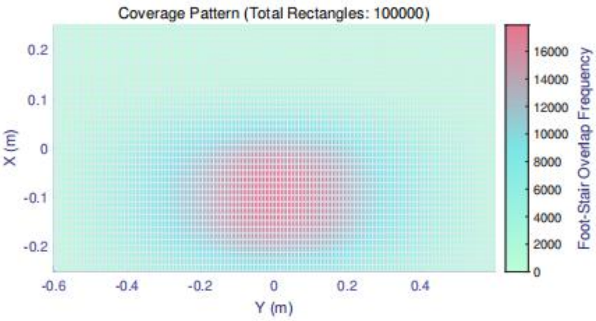

Figure 6: Walk side by side

For the scenario of side-by-side walking, this paper obtains a schematic diagram as shown in Figure 6. This is based on the process described above, combined with the judgment conditions added to the side-by-side walking model in the previous section. Similarly, when pedestrians are going up the stairs, this paper find that μ_γ = 7.62746 and σ_γ = 14.66. Additionally, the mean and variance in the x-direction remain unchanged during stair ascent and descent, with μ_x = 0 and σ_x = 183.33.

3.2. Estimation of other parameters

In addition to the several parameters mentioned above, this paper also need to determine ε_0, γ, n, Δx, Δy and the sizes of the steps.

First of all, this paper only considers the wear caused by weathering effects, so the following parameters are set in accordance with the principle of weathering. As analyzed earlier in Section 2, ε_0 represents the ideal degree of environmental weathering under initial conditions, and γ is the gradient of the amount of wear caused by environmental weathering with respect to the effect of time. Obviously, environmental factors should vary in different regions. However, the above parameters generally do not differ significantly in the differentiation models studied in this paper [10]. Therefore, these values are set as follows: $\varepsilon_0 = 0.01\text{mm} / year, n = 0.7, \gamma = 0.003$.

Next, since this paper are converting continuous steps into discrete small squares, the side lengths of these small squares also need to be determined. The smaller the size of the small squares, the closer the data is to the final result; however, when the size is too small, the number of iterations increases exponentially. Thus, in this paper, this paper set it as $\Delta x = 0.005, \Delta y = 0.01$.

Finally, this paper needs to determine the dimensions of the steps. Let e denote the length of the steps and r denote the width of the steps. In accordance with national standards [11], all the steps studied in this paper are set as $e = 1.2m, r = 0.5m$.

4. Modeling Applications and Problem Solving

4.1. Consistency of wear predictions with actual

Nondestructive testing is crucial for archaeological research and inspection, enabling rapid acquisition of sufficient sample data without disrupting the site or artifacts. Currently, archaeologists commonly use methods such as laser scanning and spectral analysis. In this work, this paper utilizes laser scanning to acquire a large amount of data. This data can be used to create an accurate 3D model of the steps and further analyze their dimensions, wear volume, and thickness. This data can also identify defects in different materials or structures, serving as a reference for renovation and restoration work. Furthermore, this paper uses a Mohs hardness scribing tool to measure the material's Mohs hardness, which can be converted and used in the model.

The paper establishes the gridded differential steps and give the walking poses. Since the footprints are rectangular and have a certain angle, the overlap with the grid is inconvenient to judge. So, this paper needs to establish an effective strategy to judge the overlap based on Separating Axis Theorem

(SAT). In the two-dimensional plane, the normal vector of each edge of a rectangle contains all the possibilities of that axis, so this paper only needs to enumerate the normal vectors of each edge of the rectangle.

After randomly generating a regular rectangular footprint, take the normal vector of one of its edges as the projection axis, project the four vertices of that rectangle and the four vertices of the grid cell rectangle onto that axis, and determine whether the projections on the axes intersect. Continue to take the normal vectors of the remaining three edges of the regular rectangle footprint as the new projection axis, and repeat the above process, as long as the regular rectangle footprint on any axis of the grid cell rectangle and the projection of the non-intersection of the situation, it is judged that the two rectangles do not overlap.

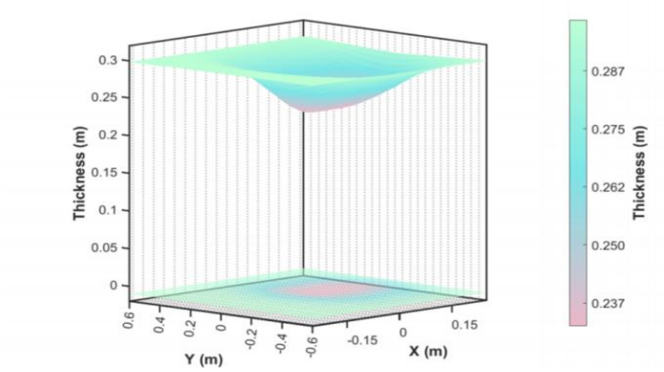

Figure 7: Step state diagram

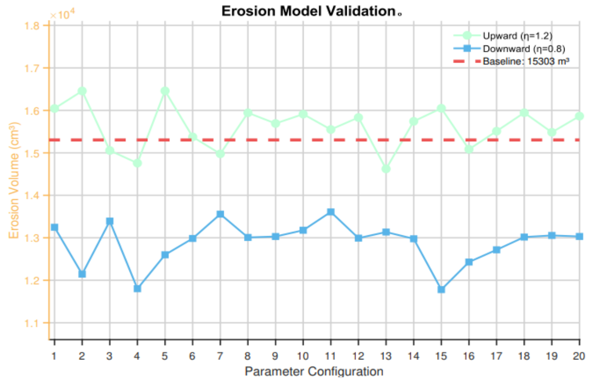

Figure 8: Line graph of total wear

In this example, this paper set $H = 300\,MPa$ and $k_0 = 0.001$. Moreover, most pedestrians walk in single file, and side-by-side walking is rare, which is also in line with local customs. Finally, all parameters in the model have been determined, and by substituting these parameters into the model, this paper obtain the total wear volume as $15540cm^3$.The difference between this result and the actual situation $15303cm^3$ proves that the model is reasonable. It can be seen from the 3D graph that the maximum wear thickness is 39.55mm, which is not significantly different from the actual maximum wear thickness of 43.24mm.Meanwhile, this paper obtained a three-dimensional map of the step state, as shown in Figure 7, which depicts the distribution of foot traffic as well as the thickness and location of the wear. This paper also analyzed the impact of upward and downward motion on the predicted results, as shown in Figure 8. In this case, this paper split the distribution to account for the majority of the population in the model moving upward or downward, and performed 20 iterations of the Monte Carlo simulation, obtaining line plots of the total amount of wear in the cases of upward and downward motion.

4.2. Signs of refurbishment or modification

Generally, the increasing wear and tear of stair treads due to human activities renders the staircase surface very uneven, affecting its normal use. In this section this paper will look at how to tell if they are refurbished and inspect them. The effect of refurbishment on the level of step wear is as follows:

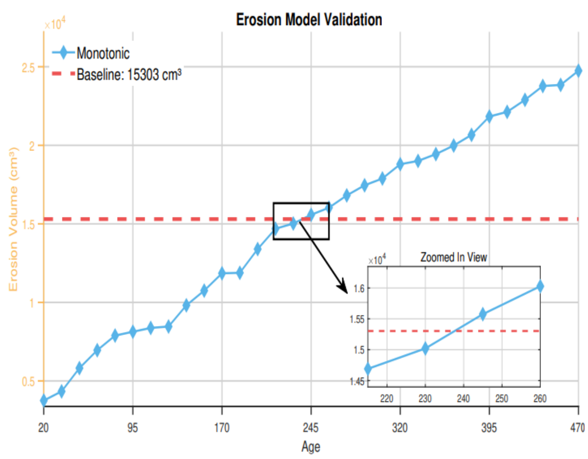

Figure 9: Gradient Search Service Life Schematic.

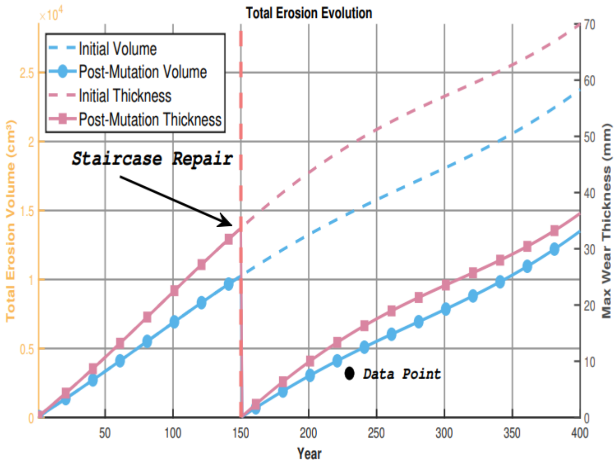

Figure 10: Plot of refurbished wear over time.

The total and maximum wear of the steps after refurbishment tends to zero, and for the sake of the aesthetics of the staircase and the functionality of the daily use, the materials used for refurbishment are almost the same as the raw materials of the steps.

Since the wear caused by human activities is much greater than that caused by environmental factors, and the wear caused by environmental weathering is smaller. Therefore, due to the process of the renovation process and other problems, the renovation can only eliminate as much as possible the wear of human activities, and the wear of environmental factors is difficult to eliminate, then the degree of wear after the renovation cannot be equal to 0.

Without considering refurbishment, the total amount of wear for a given length of use can be predicted using the model in section 3, with little difference from the actual amount of wear. However, if the predicted value is less than the actual value by more than 30 per cent, there is reason to suspect refurbishment. To prove the reliability of this suspicion, this paper found another set of stairs in Qinghai-Tibet, China, with parameters close to those of the previous example, except that they had been refurbished in a particular year.Where the black is the real total wear points.

This paper learnt from the measurements that the total amount of wear was 4437 cm3 , which was too far from the predicted value of 15540 cm3 , so the paper considered that it might have been refurbished. To further test this, this paper add refurbishment to the wear prediction model in one of the intermediate years, i.e., to reduce the wear level to be close to zero. Because the refurbishment is generally in the larger degree of wear, so the time is more backward, this paper takes the 150 years when the refurbishment, the resulting wear degree graph is shown in Figure 9.

As can be seen in Figure 10, after the inclusion of refurbishment, the difference between the total amount of wear and tear in the post refurbishment curve and the data points is very small, indicating that its actual refurbishment is actually around 150 years as well, and a search of the historical records [one] shows that the staircase was refurbished for 160 ± 6 years as measured by the historian, which illustrates the reliability of the model.

5. Conclusion

This study constructs a step surface loss model by evaluating man-made wear via Archard's equation and integrating environmental factors. Its innovation lies in combining Monte Carlo simulation with Archard's equation—leveraging mathematical statistics and probability theory—to realistically simulate random pedestrian footfall distribution. Quantitative metrics like maximum and average wear thickness provide benchmarks for inferring past pedestrian behaviors and flow.

Verification confirms the model’s validity: variable step search-estimated staircase service life aligns with measurements; renovation-integrated simulations match historical records; and wear predictions closely reflect actual values. Sensitivity analysis strengthens rigor. Though limited, the

Monte Carlo-Archard integration offers a novel quantitative approach to link step wear with pedestrian activity, addressing single-model deficiencies in complex wear simulation.

Given the complexity of inverting historical data from wear patterns and limited existing research, this model aids archaeologists. This paper welcome suggestions to enhance its applicability and accuracy.

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