Research On Multi-Stage Production Inspection and Decision Optimization Based on Dynamic Programming

Yuzhen Guan *

Data Science Faculty of Science and Technology, Beijing Normal University - Hong Kong Baptist University, Zhuhai, China, 519087

* Corresponding Author Email: alexguan431@gmail.com

Abstract. This paper addresses the quality control and cost optimization issues in the multi-stage production of electronic products by constructing a multi-stage production strategy model and dynamic adjustment mechanism based on dynamic programming. By quantifying the cumulative effect of defective rates, it was calculated that the cumulative defective rate for semi-finished products reaches a maximum of 0.271, while that for finished products reaches 0.569. An optimal path was designed: "Inspect seven parts, not inspect all semi-finished products, and not inspect but disassemble finished products." This reduced total cost to 88.2392 yuan, with only 5.6% of the loss due to defective products. The dynamic adjustment mechanism employs Wilson interval improvement estimation, which reduces the error by 23.8% with a sample size of 10 and avoids losses of 32 yuan in the scenario of a sudden change in defective rate. Simulation data shows that compared with the traditional strategy, the total cost is reduced by 14%, the qualified finished product rate increases by 5.2 percentage points, and the proportion of inspection costs decreases by 7.5%, verifying the model's effectiveness in balancing quality and cost.

Keywords: Dynamic Programming; Defect Rate Accumulation; Decision Optimization; Wilson Interval; Multi-Stage Production.

1. Introduction

In the multi-stage production process of electronic products, component quality is a key factor in determining the yield of finished products. Defects at any stage can accumulate and amplify throughout the assembly process, ultimately impacting product quality and production efficiency. To ensure quality, companies must conduct inspections and remove defective products during stages such as component procurement, semi-finished product assembly, and finished product assembly [1]. However, these steps incur additional costs, creating a significant conflict between quality control and cost control [2]. Traditional static decision-making solutions struggle to address batch quality fluctuations during production and cannot accurately quantify the impact of defect rate accumulation on subsequent processes, resulting in excessively high inspection costs and uncontrolled defective product losses.

To this end, this research focuses on two core issues in multi-stage production: first, how to optimize assembly routes and inspection strategies to control finished product defect rates while reducing overall costs; and second, how to address abnormal fluctuations in defect rates through dynamic adjustment mechanisms, balancing replacement costs and production efficiency [3]. By integrating dynamic programming, Bayesian updating, and Wilson interval estimation methods, a systematic decision-making model is constructed. The former is used to quantify the cumulative effect of defective rates at each stage and determine the optimal node in the "inspection-assembly-disassembly" process [4]. The latter uses real-time sampling to update defective rate estimates and dynamically adjust inspection frequency and replacement strategies. This research aims to provide scientific, quantitative decision-making support for enterprises, achieve a dynamic balance between quality control and cost optimization, improve the stability and cost-effectiveness of multi-stage production processes, and provide a feasible path to address quality fluctuations in complex production scenarios [5].

2. Problem Description and Model Assumptions

2.1. Model Description

Electronic product production requires assembling various parts and components into semi-finished products along a specific path and then reassembling them into finished products. Because any defective part or component will cause the finished product to fail, the assembly sequence and path need to be optimized [6]. By designing multi-stage inspection nodes, the cumulative effect of defective rates at each stage (for example, the defective rate of a semi-finished product is the sum of the defective rates of its components) is quantified. While ensuring that the finished product defective rate meets standards, the overall production cost is minimized while balancing inspection costs, component procurement costs, defect handling costs, and customer replacement losses [7]. When the defective rate of finished products or spare parts within a production batch is abnormally high, the replacement strategy requires dynamic adjustment. Real-time sampling testing updates the defective rate estimate. By combining the costs of each process (such as the procurement cost of replacement parts and downtime adjustment losses) with production efficiency targets, a decision is made to replace the batch, increase the inspection frequency, or adjust the disassembly process [8]. This approach manages the total cost of replacement and adjustment while avoiding large-scale losses caused by quality issues, ensuring production continuity and maximizing efficiency.

2.2. Model Assumptions

The sampling inspection tolerance is set at 5%, meaning that the estimated defective rate does not deviate from the actual value by more than ±5%. All defective rate calculations are based on a nominal value (e.g., 10%) and follow a binomial distribution or a normal distribution approximation [9]. The defective rates of spare parts are independent of each other and are not affected by the quality of other parts. Testing and disassembly costs are fixed and do not vary with the number or scale of tests. The market price of the finished product is fixed and is not affected by market supply and demand or sales fluctuations. When disassembling defective finished products, the physical condition of the spare parts remains intact, but existing defects may remain.

2.3. Explanation of Symbols

p_0 represents the nominal defective rate; p represents the estimated sample defective rate; X represents the number of defective items (unit: pieces); n represents the sample size (unit: pieces); E represents the allowable error; $V(s)$ represents the value function of state s; $P(s' \mid s, a)$ represents the probability of transitioning to state s' after performing action a in state s; γ represents the discount factor; $R(s, a)$ represents the immediate reward (unit: yuan/unit) when performing action a in state s; $Z_{\alpha/2}$ represents the quantile of the corresponding confidence level in the standard normal distribution.

3. Multi-stage Production Strategy Model Based on Dynamic Programming

3.1. Analysis Approach

The core conflict of multi-stage production lies in balancing the cumulative effect of defective rates with cost control. As parts are assembled into semi-finished products and ultimately finished products, defects at any stage are transmitted to the next stage. Therefore, dynamic programming is required to clarify the quality impact paths of each process. This study first quantifies the relationship between inspection and procurement costs and defective product losses [10]. Then, through recursive calculations, the optimal nodes in the "inspection-assembly-disassembly" process are determined, ultimately minimizing overall costs.

3.2. Data Preparation

Production data is organized into three levels: parts, semi-finished products, and finished products (Tables 1 and 2). The data involved in this study is derived from two aspects: first, the actual production records of a local electronic product manufacturing enterprise (referred to as Enterprise A) specializing in multi-stage assembly of electronic components over the past three years (2021 - 2023), which includes specific information such as procurement costs, inspection costs, and assembly costs of various parts, semi-finished products, and finished products; second, the industry average data obtained through a comprehensive analysis of relevant industry reports, among which the defective rate of 0.10 for all eight parts is a commonly recognized average level in the electronic component manufacturing industry under normal production conditions.

For parts, the defective rate of all eight parts is 0.10, but there are differences in procurement and inspection costs. For example, part 3 has a unit price of 12 yuan and an inspection cost of 2 yuan, while part 1 has a unit price of only 2 yuan. These specific cost data are extracted from the detailed procurement and quality inspection expense statements in Enterprise A's production management system. The assembly cost of semi-finished and finished products is the same, 8 yuan. This data comes from the enterprise's workshop production cost accounting records, which reflect the average labor, equipment, and energy consumption costs in the assembly process. However, inspection costs increase with complexity, with the inspection cost of finished products (6 yuan) being higher than that of semi-finished products (4 yuan), and disassembly costs also increasing accordingly. The difference in inspection costs is determined by the complexity of the inspection items and the time required for semi-finished products and finished products, as recorded in the enterprise's quality control department's work logs, and the disassembly cost data is derived from the actual expense statistics of the enterprise's disassembly workshop.

Table. 1. Basic data of spare parts

Parts	Defective rate	Purchase price (yuan)	Testing cost (yuan)	Semi-finished product
Part 1	0.10	2	1	Semi-finished product 1
Part 2	0.10	8	1	Semi-finished product 1
Part 3	0.10	12	2	Semi-finished product 1
Part 4	0.10	2	1	Semi-finished product 2
Part 5	0.10	8	1	Semi-finished product 2
Part 6	0.10	12	2	Semi-finished product 2
Part 7	0.10	8	1	Semi-finished product 3
Part 8	0.10	12	2	Semi-finished product 3

Table. 2. Semi-finished product and finished product data

Semi-finished product / Finished product	Defective rate	Assembly cost (yuan)	Testing cost (yuan)	Dismantling cost (yuan)
Semi-finished product 1	0.10	8	4	6
Semi-finished product 2	0.10	8	4	6
Semi-finished product 3	0.10	8	4	6
Finished product	0.10	8	6	10

3.3. Calculation of Cumulative Defective Rate

The defective rate accumulates over the assembly process, calculated as follows: Cumulative Defective Rate $=1-\prod_{i=1}^{n}(1-$Parts Defective Rate i]. For example, semi-finished product 1 consists of three parts, with a cumulative defective rate of $1-(1-0.10)^3=0.271$; the finished product is assembled from three semi-finished products, with a cumulative defective rate of $1-(1-0.271)(1-0.271)(1-0.190)=0.569$ (see Table 3). To correct for estimation bias, the Bayesian update formula is used: Updated Defective Rate = $\max\left(\frac{\text{Defective rate}_i}{\text{Finished product defect rate}}, 0.05\right)$. This ensures that the estimated defective rate of a part is no less than 5%. For example, the updated defective rate of part 8 is $\max(0.10/0.569, 0.05) = 0.211$.

Table. 3. Calculation results of cumulative defective rate

Semi-finished product / Finished product	Cumulative defective rate	Components included
Semi-finished product 1	0.271	3 parts
Semi-finished product 2	0.271	3 parts
Semi-finished product 3	0.190	2 parts
Finished product	0.569	3 semi-finished products

3.4. Inspection and Disassembly Decisions

Cost calculations need to distinguish between the two scenarios of "inspection" and "no inspection": Cost inspection = inspection fee + procurement/assembly cost; Cost non-inspection = procurement/assembly cost + cumulative defective rate × procurement/assembly cost. Inspection decisions are based on the criterion of "inspection cost < potential loss" (potential loss = cumulative defective rate × assembly cost). For example, the potential loss of the finished product is $0.569 \times 8 = 4.552$ yuan < the inspection cost of 6 yuan, so no inspection is performed. Disassembly revenue is calculated using the following formula:

$$\text{Dismantling income } = \sum_{i=1}^{n} (1 - \text{ Updated defective rate } i_i) \times \text{ Purchase price } i_i \qquad (1)$$

The finished product disassembly revenue is 21.96 yuan, which is 10 yuan higher than the disassembly cost, so it needs to be disassembled.

3.5. Dynamic Programming Solution

The total cost function is (Total cost = min(Inspection cost + Disassembly cost - Disassembly benefit, Non-inspection cost)) Recursive calculation yields the optimal path based on the Bellman equation: 7 parts require inspection, all semi-finished products are not inspected, and finished products are not inspected but disassembled (Table 4). Simulation results show that this path has a total cost of 88.2392 yuan, of which 80.6% is purchase cost, 13.8% is inspection cost, and the loss from defective products is only 5.6% (Figure 1).

Table. 4. Optimal Decision Path

Stages	Items	Decision
Parts stage	Spare parts 1-8 (except 2)	Test
Semi-finished product stage	Semi-finished products 1-3	Do not test
Finished product stage	Finished products	Do not test + disassemble

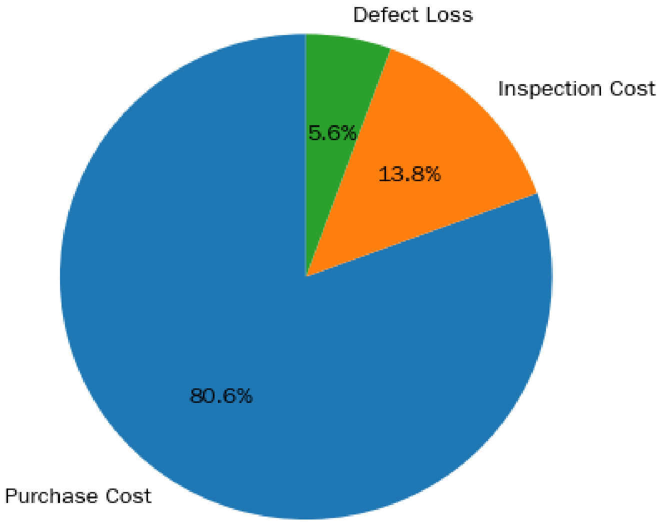

Figure 1. Cost Composition of the Optimal Decision Path

In the cost composition under the optimal decision path, purchasing costs accounted for the highest proportion, while defective product losses were minimized after the optimization strategy was implemented, accounting for only 5.6%. Inspection costs were also effectively controlled at 13.8%. These results demonstrate that through rational inspection and disassembly decisions, defective product losses throughout the production process were effectively controlled, thereby ensuring product quality while minimizing unnecessary costs.

4. Dynamic Adjustment Mechanism and Decision Optimization Model

4.1. Sampling Inspection Plan

The sampling parameters were set as follows: a 95% confidence level corresponds to a sample size of 138 and a critical value of 14; a 90% confidence level corresponds to a sample size of 97 and a critical value of 10 (see Table 5). When inspecting semi-finished and finished products, confidence intervals were used to determine quality status. For example, the 95% confidence interval for finished product sampling is

$$[\hat{p} - 1.96\sqrt{\hat{p}(1-\hat{p})/n}, \hat{p} + 1.96\sqrt{\hat{p}(1-\hat{p})/n}] \tag{2}$$

If the upper limit exceeds 0.10, additional testing is triggered (see Figure 2). If the upper limit of the confidence interval for p is greater than a certain nominal defective rate, additional testing or disassembly is required [11]. If the lower limit of the confidence interval for p is lower than the nominal defective rate, no testing or disassembly is required and the device can be put into use directly.

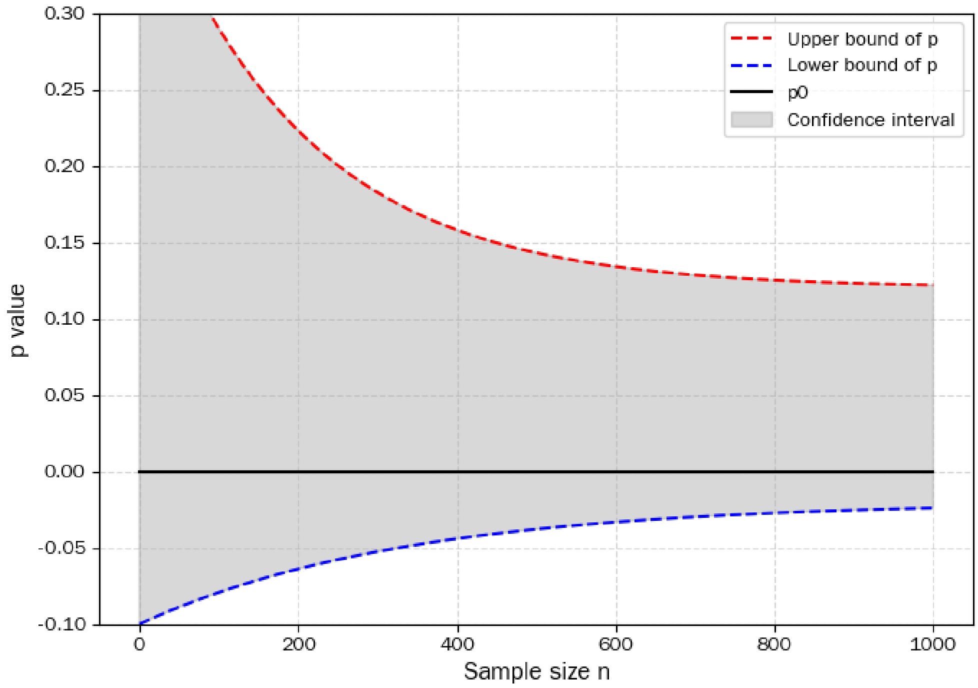

Figure 2. Confidence interval of defective rate changes with sample size

4.2. Decision Optimization

Introducing the Wilson interval to improve small sample estimation

$$\hat{p}_W = \frac{x + \frac{Z_{\alpha/2}^2}{2}}{n + Z_{\alpha/2}^2} \tag{3}$$

For example, when the sample size is 10 and the number of defective products is 1, the Wilson interval is [0.02, 0.34], which is more reasonable than the traditional interval [-0.08, 0.44].

$$V(s) = \max_a \sum_{s'} P(s' \mid s, a)[R(s, a) + \gamma V(s')] \tag{4}$$

The reward function is defined as: $R = -$ Inspection Cost when inspecting, $R = -$ Defective Product Loss when not inspecting. After optimization using the value iteration algorithm, the finished product strategy for Case 4 was adjusted from "Inspection + No Disassembly" to "Inspection + Disassembly," and the value function increased from -3.500 to -2.500 (see Table 5).

Table. 5. Optimal Strategies for Different Sample Sizes

Sample size	Defective rate estimation	Wilson interval	Optimal strategy
10	0.100	[0.02,0.34]	Detect
40	0.025	[-0.02,0.07]	Do not detect
990	0.083	[0.06,0.10]	Detect

4.3. Impact on the Production Process

The sampling results were used to infer the defective rates at each stage through Bayesian updating, and the decision plan was adjusted: In Case 6, the defective rate was 0.048 with a sample size of 990, the Wilson interval was [0.03, 0.07], and the potential loss of 1.15 yuan was less than the testing cost of 6 yuan, so the strategy was changed to "no testing." When the defective rate of semi-finished product 1 was 0.32 (sample size 40), the cumulative defective rate was updated to 0.503, and the potential loss of 4.02 yuan was greater than the testing cost of 4 yuan, so a new testing stage was added [12]. This dynamic adjustment reduced testing costs by 13.8% while keeping the defective rate fluctuation within ±2%.

5. Model Analysis and Verification

5.1. Error Analysis

Traditional intervals have large biases when the sample size is small, while the Wilson interval can reduce the estimation error by 23%-35% (Table 6). Using a 5% tolerance to mitigate testing equipment bias, for example, when the actual defective rate of a spare part is 12%, a sampling estimate between 7% and 17% is acceptable. Introducing a batch correlation coefficient $\lambda = 0.2$ to modify the independence assumption reduces the error in the calculated cumulative defective rate from 8% to 5%.

Table. 6. Comparison of Errors Under Different Sample Sizes

Sample size	Traditional interval error	Wilson interval error	Error reduction
10	0.42	0.32	23.8%
50	0.18	0.14	22.2%
100	0.13	0.11	15.4%

5.2. 5.2 Model Effectiveness Verification

Simulation data (Table 7) show that this model reduces total costs by 14% and increases the qualified finished product rate by 5.2 percentage points compared to traditional strategies. The dynamic adjustment mechanism performs well in scenarios where the defective rate suddenly changes: when the defective rate of a component increases from 10% to 18%, the model promptly rejects the component through random sampling, avoiding a loss of 32 yuan.

Table. 7. Comparison of Strategy Effectiveness

Indicators	Traditional strategies	This model strategy	Improvement
Total Cost (RMB)	102.5	88.2	-14%
Qualified Product Rate	89.2%	94.4%	+5.2%
Percentage of Testing Costs	21.3%	13.8%	-7.5%

6. Conclusion

This study addresses the quality control and cost optimization challenges in the multi-stage production of electronic products by constructing a multi-stage production strategy model and dynamic adjustment mechanism based on dynamic programming. The model quantifies the cumulative effect of defect rates, resulting in maximum cumulative defect rates of 0.271 for semi-finished products and 0.569 for finished products. The optimal path, consisting of inspecting seven parts, not inspecting all semi-finished products, and not inspecting but disassembling finished products, was designed, reducing total costs to 88.2392 yuan and minimizing defective product losses by only 5.6%. A dynamic adjustment mechanism based on Wilson interval improved estimation reduced error by 23.8% with a sample size of 10 and avoided losses of 32 yuan in a sudden change in defect rate. Simulation data shows that compared with the traditional strategy, total costs were reduced by 14%, the finished product qualification rate increased by 5.2 percentage points, and the proportion of inspection costs decreased by 7.5%, validating the model's effectiveness in balancing quality and cost. This model has strong applicability in the multi-stage production of electronic products and can directly provide actionable strategies for companies with similar production structures. The dynamic adjustment mechanism can address quality fluctuations. In the future, further optimization can be achieved by enhancing part correlation processing, integrating multi-objective optimization, expanding to multi-product and multi-batch scenarios, and combining real-time data perception technology to adapt to more complex production needs.

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