Product Production Decision Model Based on Cost Minimization

Gongle Shen [1], Chun He [2, *]

[1] School of Materials and Energy, Guangdong University of Technology, Guangzhou, China, 510006

[2] Department of Mathematics and Statistics, Guangdong University of Technology, Guangzhou, China, 510006

* Corresponding Author Email: qishen179@163.com

Abstract. To achieve maximum profit, controlling costs during the production process has become a top priority in enterprise production planning. A certain enterprise needs to manufacture and sell a finished product by purchasing different spare parts and assembling them in order to produce an electronic product. This article establishes a product production decision model based on cost minimization, which helps enterprises make the most reasonable production decisions and achieve maximum profits. This article establishes a decision-making model for the sampling and testing plan of spare parts based on hypothesis testing method. According to the central limit theorem in statistics, the original distribution is followed when the sample size is small, while the normal approximation can be used instead of the original distribution when the sample size is large. The calculation result is as follows: In (1), if the sample size n=29, the enterprise should reject this batch of spare parts when the number of defective products reaches 6; If the sample size n=1000, the enterprise should reject this batch of spare parts when the number of defective products reaches 116. In (2), if the sample size n=29, the enterprise should reject this batch of spare parts when the number of defective products reaches 5; If the sample size n=1000, the enterprise should reject this batch of spare parts when the number of defective products reaches 113.This article transforms the problem of maximizing final net profit into the problem of minimizing final cost value, and establishes a product production decision model based on the minimum cost value. This article mathematizes the five decision points that exist in the production process and sets corresponding variable decision variables n=1,2,..., 5, x_n values of 1 or 0 to express whether the decision is to be executed or not. After obtaining all decision options through enumeration, calculate the cost value corresponding to each option, and take the decision option corresponding to the lowest cost value as the best decision option in this situation.

Keywords: Statistics, Hypothesis testing, Central limit theorem, Normal approximation, Z-test.

1. Introduction

In the actual production and operation process, enterprises need to inspect the quality of the purchased goods. We need to design a testing plan with as few sampling times as possible to check whether the supplier's spare parts meet their claimed nominal values.

Zheng Zhaocong et al. (2020) established a barcode risk balanced sampling model, which resulted in a false detection rate of ↓ 18% and an efficiency of ↑ 35% [1]; Liang Kaikai (2023) proposed an innovative construction method for online retail commodity price index based on crawler data sampling, which increased data coverage by 42% and computational timeliness by 28%, solving the pain points of traditional compilation [2]; Suh et al. (2014) developed the OpenSample dynamic sampling platform to achieve SDN microsecond level latency monitoring [3].

Based on previous research, scholars have not found a suitable method to solve the problem of product sampling during enterprise procurement.

And this article designs a model to apply to this problem, solving the product sampling problem when enterprises purchase goods. We assume that there are only two states for spare parts: genuine and defective, which meet the conditions of binomial distribution. According to the central limit theorem, when the sample size is large enough, the binomial distribution can be approximated as normal. Based on this, we can divide product sampling into two situations: small sample size and large sample size, and solve them using binomial distribution and normal approximation, respectively.

In order to better describe the proposed cargo sampling and inspection scheme in this article, it is divided into five parts. The first chapter introduces the relevant background of the product sampling problem when the enterprise purchases goods established in this article. The second part introduces the relevant theories of the model, such as binomial distribution, normal distribution, and the De Moivre Laplace central limit theorem. The third part introduces how the model is established in this article. The fourth part introduces the conclusions obtained after establishing the model in this article. The fifth part summarizes the model established in this article and verifies the conclusions we have obtained.

2. Research on Supplier's Goods Based on Sampling Inspection Method

2.1. Binomial distribution

If the probability function of the random variable X is $P\{X = x\} = \binom{n}{x} p^x (1-p)^{n-x}, x = 0,1,\dots,n$, then X is said to follow a binomial distribution with parameters n and p, denoted as $X \sim b(n,p)$ [4]. The standard error SE is

$$SE = \sqrt{p_0(1-p_0)/n} \tag{1}$$

The formula for the cumulative distribution function of a binomial distribution is

$$P\{X \le x\} = \sum_{k=0}^{x} \binom{n}{x} p^k (1-p)^{n-k} \tag{2}$$

The schematic diagram of binomial distribution is shown in Figure 1.

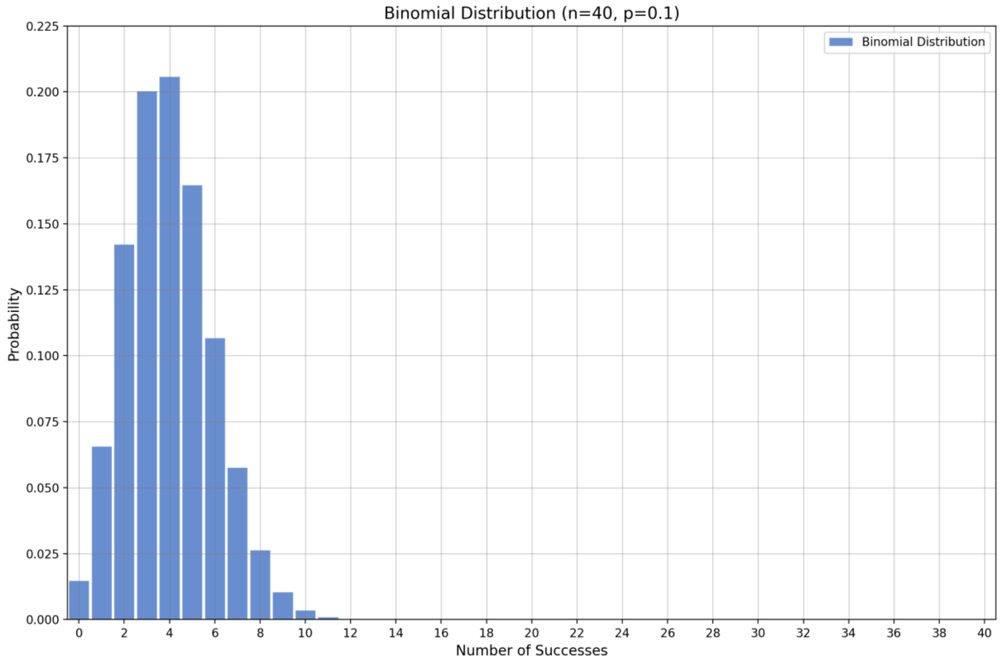

Figure 1. Schematic diagram of binomial distribution

2.2. Normal distribution

If the density function of a continuous random variable X is $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, -\infty < x < +\infty$, where μ, σ are constants, then X is said to follow a normal distribution with parameters μ, σ, denoted as $X \sim N(\mu, \sigma)$ [5]. The formula for its cumulative probability function is $P\{X \le x\} = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{x} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx$.The schematic diagram of normal distribution is shown in Figure 2.

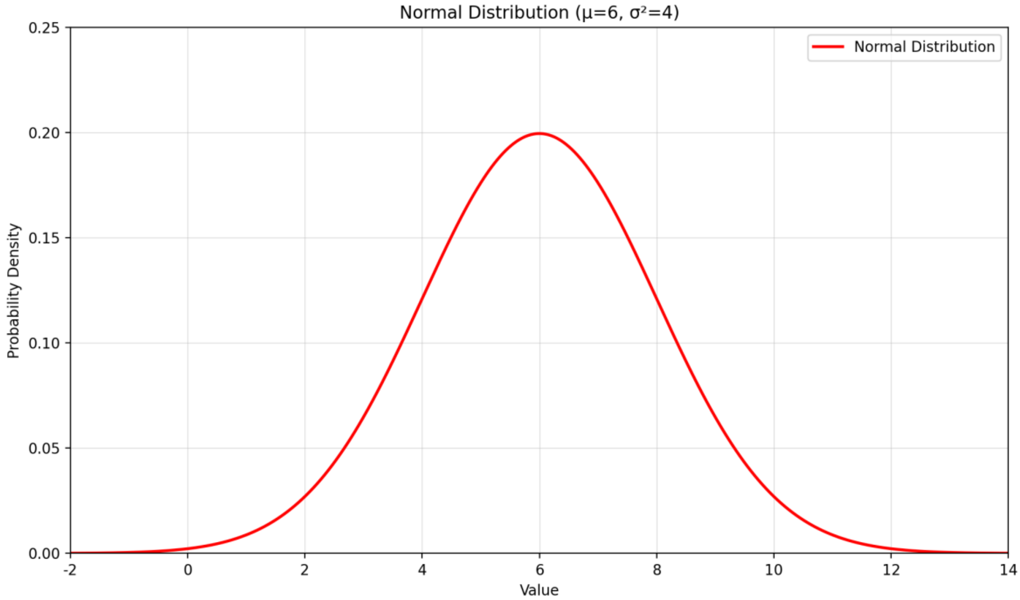

Figure 2. Schematic diagram of normal distribution

2.3. De Moivre Laplace Central Limit Theorem

The Dumont Laplace central limit theorem is the initial version of the central limit theorem, which states that the binomial distribution has a normal distribution as its limit. When the sample size is large enough, the binomial distribution can be approximated as a normal distribution, so this article will use a normal distribution instead of the binomial distribution in calculations with large sample sizes.

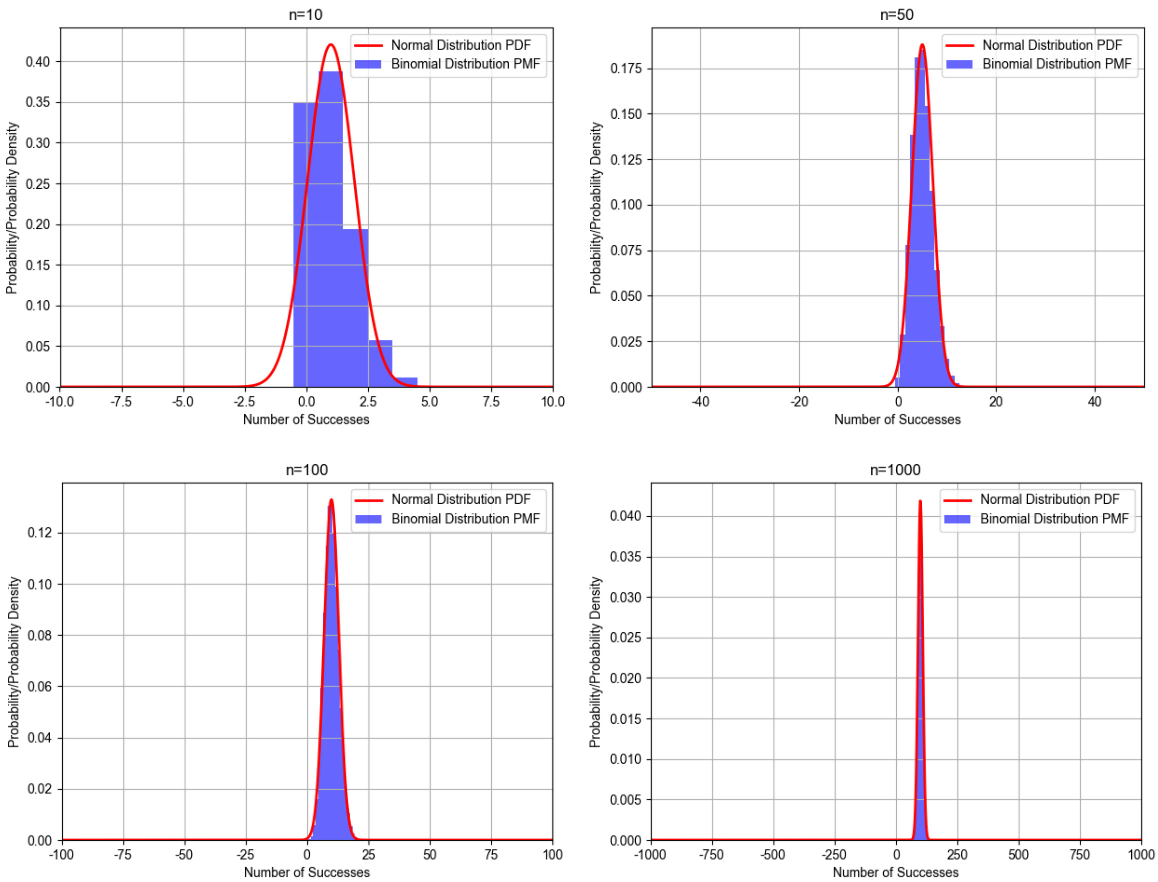

Figure 3. Comparison between binomial distribution and normal distribution for n=10, 50, 100, 1000

As shown in Figure 3, as the sample size continues to increase, the binomial distribution gradually approaches a normal distribution.

2.4. Model establishment

When the population and sample size are small, the binomial distribution cannot be approximated as a normal distribution, and the Z-test is not applicable to this condition. So we use a binomial distribution for precise sampling and testing plan design, using the probability cumulative formula (3)

$$P\{X \le x\} = \sum_{k=0}^{x} \binom{n}{x} p^k (1-p)^{n-k}, x \le 30 \tag{3}$$

Calculate the critical value Z_α for this reliability level. If the enterprise detects the number of defective products corresponding to the critical value Z_α, it can reject the null hypothesis H_0 and refuse to accept this batch of spare parts.

When the population and sample size are large enough, the binomial distribution can approximate a normal distribution, and the Z-test is applicable. We use the Z-test method in hypothesis testing based on the normal and normal approximation to design a sampling testing plan. The calculation formula for the actual defect rate $\hat{p}$ derived from (8) is

$$\hat{p} = Z \cdot \sqrt{\frac{p_0(1-p_0)}{n}} + P_0 \tag{4}$$

From the first type of error, it can be inferred that the significance level α is 1-reliability. Therefore, based on the reliability given in the question, the corresponding critical value statistic Z_α can be obtained. Substituting it into (16) can calculate the actual defect rate $\hat{p}$ corresponding to the critical value, and substituting it into the formula for calculating the number of defective products $n_{defective\ product}$.

$$n_{defective\ product} = n \cdot \hat{p} \tag{5}$$

2.5. Model solution

(1) At 95% confidence:

Assuming null hypothesis H_0.

$$H_0: p \le p_0 \tag{6}$$

Alternative assumption H_1.

$$H_1: p > p_0 \tag{7}$$

The significance level $\alpha = 1 - 0.95 = 0.05$ and statistical efficacy $1 - \beta$ are assumed to be 80%, i.e. $\beta = 0.20$. This value is widely accepted as a sufficiently high efficacy level to ensure high reliability of the study.

The $n \le 30$ sample size n is small, and an accurate test of binomial distribution is used. After Python calculation, when the sample size is n=29, the critical value of 95% reliability corresponds to 6 defective products. That is, when 6 defective products are detected, the null hypothesis is rejected, and it is considered that the defect rate of this batch of spare parts is greater than 10%. It is recommended that the enterprise reject this batch of spare parts [6].

The $n > 30$ sample size n is large enough to use normal approximation instead of binomial distribution, and it meets the conditions for using Z-test in hypothesis testing. Using Python programming calculation, when the sample size is n=1000, the critical value of 95% reliability corresponds to 116 defective products, which means that when 116 defective products are detected, the null hypothesis H_0 is rejected, as shown in Figure 4.

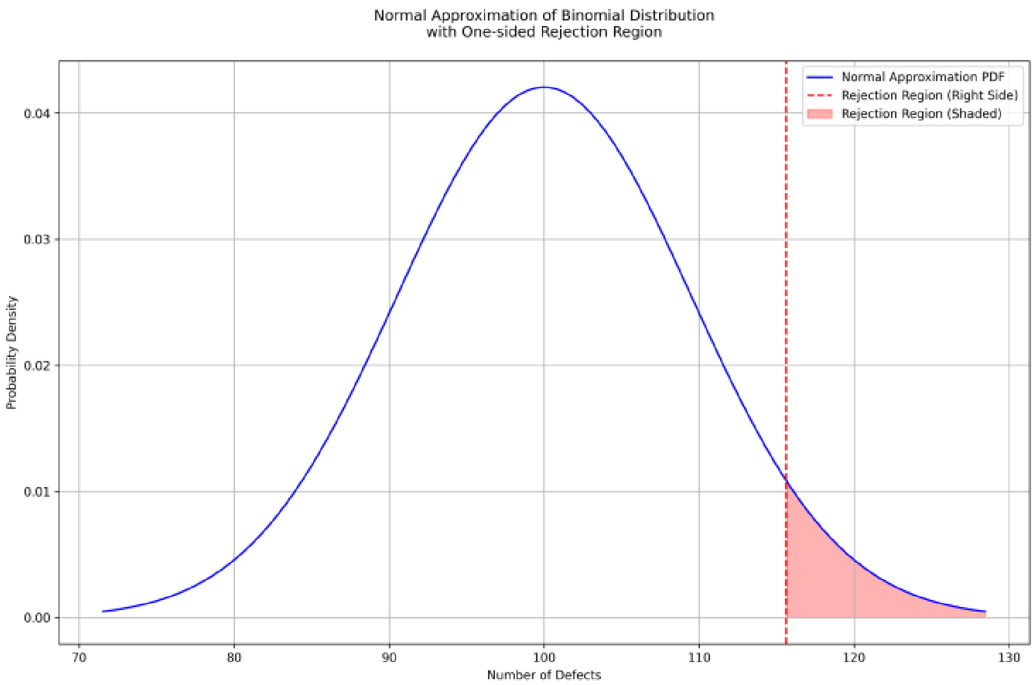

Figure 4. 95% confidence, n=1000, positive too approximate rejection domain

For (2) at a 90% confidence level, the normal distribution diagram is shown in Figure 5:

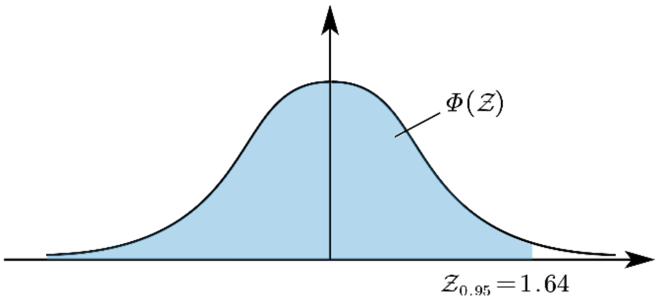

Figure 5. 95% reliability normal distribution diagram

For (2) at a 90% confidence level, the normal distribution diagram is shown in Figure 6~Figure 8:

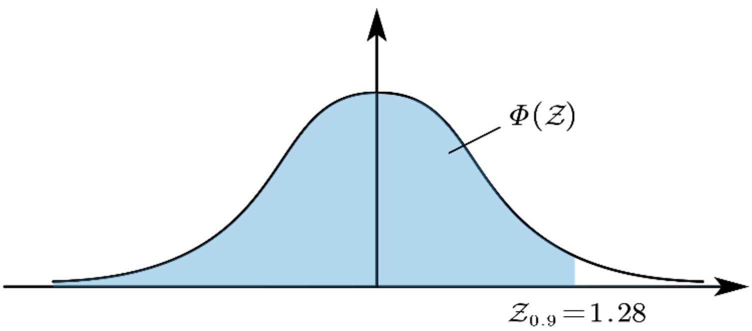

Figure 6. Normal distribution of 90% reliability

Assumption H_0

$$H_0: p \le p_0 \tag{8}$$

Alternative hypothesis H_1

$$H_1: p > p_0 \tag{9}$$

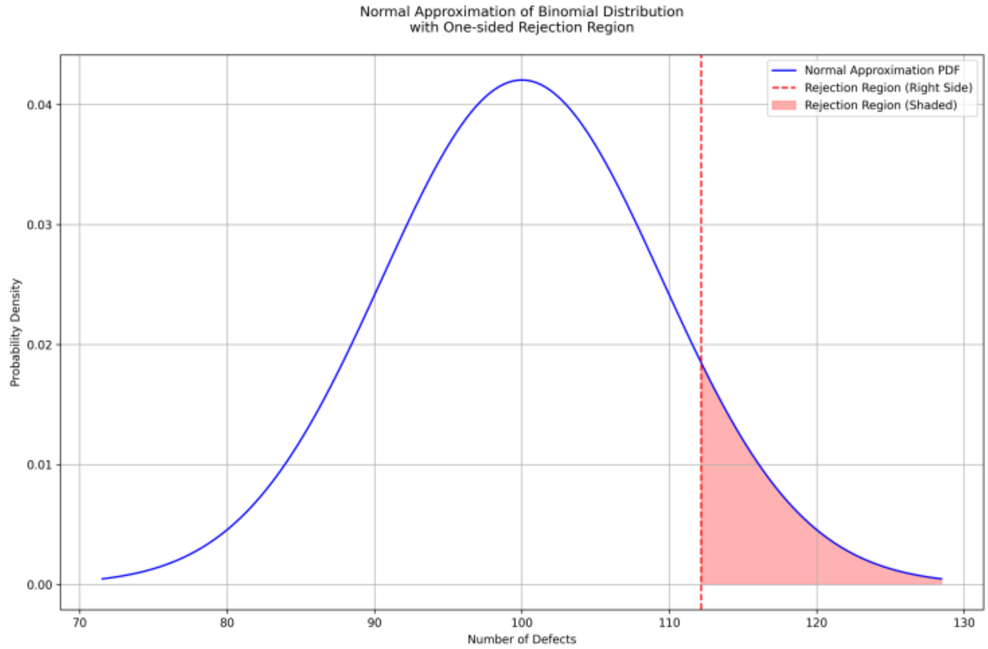

Figure 7. The rejection domain of the normal approximation with 90% reliability and n=1000

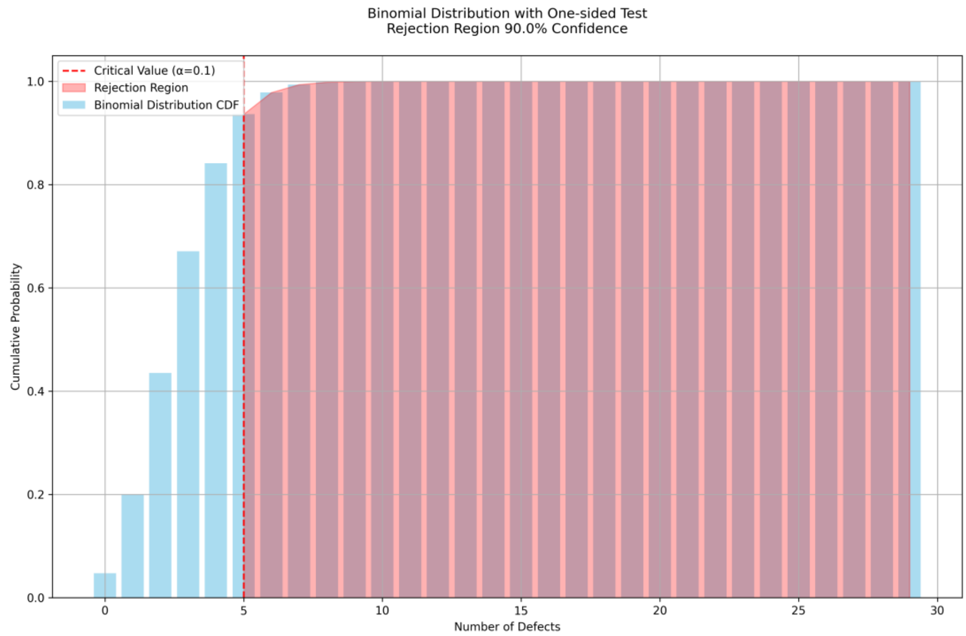

Figure 8. Refusal Domain of Binomial Distribution with 90% Reliability and n=29

The significance level $\alpha = 1 - 0.90 = 0.10$ and statistical efficacy $1 - \beta$ are assumed to be 80%, i.e. $\beta = 0.20$. This value is widely accepted as a sufficiently high efficacy level to ensure high reliability of the study [7].

$n \leq 30$: The sample size n is small, and a precise test of binomial distribution is used. After Python calculation, when the sample size is n=29, the critical value of 90% reliability corresponds to 5 defective products. That is, when 5 defective products are detected, the null hypothesis is rejected, and it is considered that the defect rate of this batch of spare parts is greater than 10%. It is recommended that the enterprise reject this batch of spare parts.

$n > 30$: The sample size n is large enough to use normal approximation instead of binomial distribution, and it meets the conditions for using Z-test in hypothesis testing. Using Python programming calculation, when the sample size is n=1000, the critical value of 95% reliability corresponds to 116 defective products, which means that when 116 defective products are detected, the null hypothesis H_0 is rejected.

3. Research on Maximizing Enterprise Profits

Enterprises are economic organizations engaged in production and business activities for profit, providing goods and services to society [8]. Therefore, in the process of enterprise production, maximizing the final net profit is an important factor that needs to be considered in every decision-making step [9]. The formula for calculating a company's profit is profit=selling price – cost [10]. Since each situation in this question corresponds to a unique finished product selling price, we transform the problem of maximizing the final net profit into pursuing the minimum final cost value, and based on this, establish the commodity production decision model in this article.

Inspection of spare parts 1, inspection of spare parts 2, inspection of finished products, dismantling of non-conforming products, and dismantling of non-conforming products returned after sale. Set the variable x_n, n=1,2,3,4,5, respectively, to control whether to detect spare part 1, spare part 2, finished product, dismantled and detected non-conforming products, and dismantled and returned non-conforming products after sale. xn=1 represents a decision of yes, xn=0 represents a decision of no.

Assuming the defect rate of part 1 is P_1, the defect rate of part 2 is P_2, the purchase of part 1 is g_1, the purchase of part 2 is g_2, the testing cost of part 1 is j_1, the testing cost of part 2 is j_2, the purchase quantity of parts 1 and 2 is N_0, the assembly cost of finished products is z, and the dismantling cost of unqualified finished products is h The exchange loss of non-conforming products after sale is d. Based on basic economic knowledge, it can be concluded that:

purchase cost C_0:

$$C_0 = N_0(g_1 + g_2) \tag{10}$$

Testing cost for parts 1 and 2:

$$C_1 = N_0(x_1 j_1 + x_2 j_2) \tag{11}$$

Quantity of finished product assembly:

$$N_1 = \min\{N_0[x_1(1 - p_1) + 1 - x_1], N_0[x_2(1 - p_2) + 1 - x_2]\} \tag{12}$$

The cost of manufacturing finished products and testing:

$$C_2 = zN_1 + x_3 j_3 N_1 + x_3[x_4 p_3 h N_1 - x_4 p_3(g_1 + g_2)] \tag{13}$$

Cost of replacing defective products without testing finished products:

$$\begin{aligned} C_3 = N_1(1 - x_3)[p_3 + (1 - x_1)p_1 + (1 - x_2)p_2][d + x_5 h - x_5(g_1 + g_2) \\ +(g_1 + g_2 + z + x_1 j_1 + x_2 j_2)] \end{aligned} \tag{14}$$

Total cost:

$$C = C_0 + C_1 + C_2 + C_3 \tag{15}$$

The product production decision model has been established.

3.1. Model solution

Based on the analysis in the previous section, it is known that there are 5 variable decision variables x_n in this article, n=1, 2, 3, 4, and 5 controlling various decisions in the production process of the enterprise. Therefore, 2^5=32 different production decision schemes can be enumerated for each situation.

In this article, all possible solutions were simulated in a computer using Python, with a purchase quantity of N_0=10000. By substituting the six scenarios in Table 1, the cost values corresponding to different solutions in each scenario can be calculated. Take the decision solution with the minimum cost value from the traversal results as the optimal decision solution.

The optimal decisions corresponding to the six production scenarios are shown in Table 1

Table 1. Calculation Results and Decision Proposal

situation	decision variable $(x_1, x_2, x_3, x_4, x_5)$ value retrieval	Decision Explanation	Cost (yuan/piece)
1	(0, 0, 1, 1, 0)	Parts 1 and 2 are not inspected, finished products are inspected, defective products detected are dismantled, and replacement defective products are not dismantled	29.3
2	(0, 0, 1, 1, 0)	Parts 1 and 2 are not inspected, finished products are inspected, defective products detected are dismantled, and replacement defective products are not dismantled	27.6
3	(0, 0, 1, 1, 0)	Parts 1 and 2 are not inspected, finished products are inspected, defective products detected are dismantled, and replacement defective products are not dismantled	29.3
4	(0, 0, 1, 1, 0)	Parts 1 and 2 are not inspected, finished products are inspected, defective products detected are dismantled, and replacement defective products are not dismantled	26.6
5	(0, 1, 1, 1, 0)	Spare parts 1 are not inspected, spare parts 2 are inspected, finished products are inspected, defective products measured are disassembled, and defective products exchanged are not disassembled	28.04
6	(0, 0, 1, 0, 0)	Parts 1 and 2 are not inspected, finished products are inspected, and defective products detected and exchanged are not disassembled	31.0

Create a finished product production cost chart corresponding to all six scenarios.

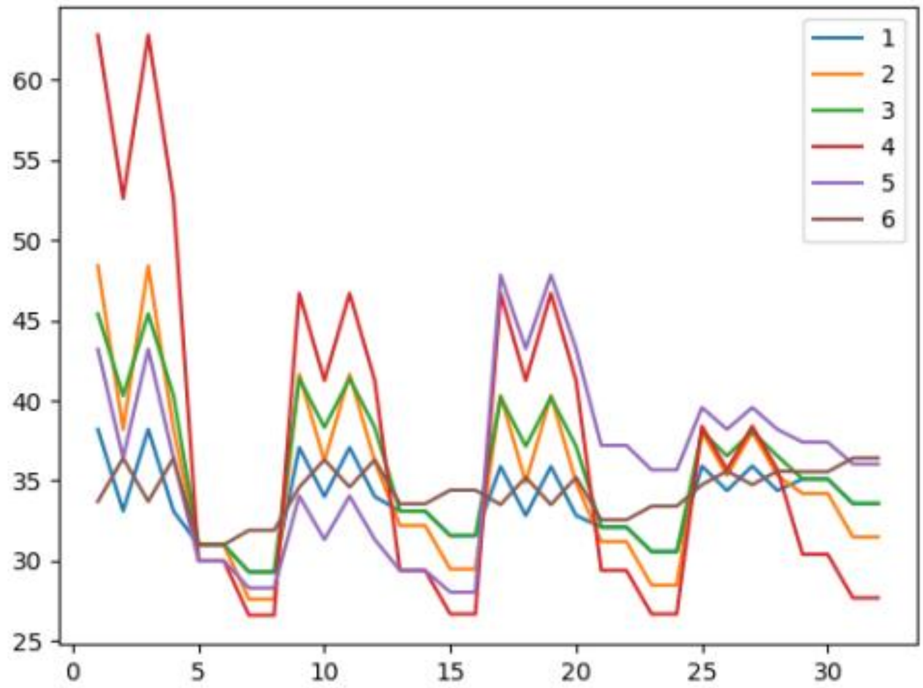

Figure 9. Cost schematic diagram of various production decision-making schemes for enterprises

4. Conclusions

This article establishes a sampling testing model based on hypothesis testing, and calculates the critical values for the number of rejected defective products for different sample sizes (using binomial distribution for small samples and normal approximation for large samples) and confidence levels (90% and 95%). The results indicate that the higher the reliability, the more defective products are required for rejection, and the inspection is more rigorous. Then, with the goal of minimizing costs, a production decision model containing 5 decision variables is established, and the optimal decision is determined by enumerating all strategy options. The results show that in most cases, the best strategy is to not inspect spare parts, only inspect finished products and dismantle the defective products detected, effectively controlling costs and providing scientific decision-making basis for enterprises.

References

[1] Zheng Zhaocong, Wu Weiqiang. Comparison of advantages and disadvantages of sampling scenarios for commodity bar code quality sampling and suggestions for selection [J]. Bar code and information system, 2020, (02) : 33-36.

[2] Liang Kaikai. Research on the construction method of online retail commodity price index based on crawler data sampling [D]. Henan University of Technology, 2023.

[3] Suh J , Kwon T , Dixon C ,et al.OpenSample: A Low-Latency, Sampling-Based Measurement Platform for Commodity SDN [J].IEEE, 2014.

[4] Zhao Liqun, Tian Yanan, Cao Lingling, et al. Study on the strong seedling index and rapid sampling detection of downy mildew of cucumber plug seedlings in Beijing, Hebei and Shandong [J]. China Agricultural Bulletin, 2021,37 (19) : 47-51.

[5] Chen M , Chen S , Xie K ,et al.A Commodity Recognition Model Under Multi-Size Lifting and Lowering Sampling [J].Electronics (2079-9292), 2025, 14 (11).

[6] Gao S , Wang S , Wang Y ,et al.ChatGPT and Commodity Return [J].Journal of Futures Markets, 2025, 45(3).

[7] Yang Xue, Liao Yanyu, Zhang Song, et al. Research on spatial balanced sampling design under the background of multi-objective survey [J]. Statistics and decision-making, 2025,41 (14) : 41-46.

[8] Sun Mingwei. Quantitative packaging commodity net content measurement test [J]. Brand and standardization, 2025, (02) : 205-207.

[9] Zhou Zulian.Swedish inspectors ' guidance on sampling inspection of pre-packaged goods [J]. Weighting instrument, 2021,50 (07) : 47-51.

[10] Ren Hao, Liu Xuwang.Standardization of commercial coal sampling inspection [J]. Modern industrial economy and informatization, 2021, 11 (03) : 160-161.