Research on Dynamic Modeling of Multibody Systems Based on Equiangular Helix Motion

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Abstract. Focusing on the motion characteristics of multi-body equidistant spiral motion, this paper takes the "bench dragon", a traditional folk activity in the Zhejiang-Fujian region, as an example to establish a dynamic mathematical model based on the equidistant spiral trajectory. It realizes numerical solution and analysis through MATLAB, and explores the motion trajectory of complex multi-body joint motion systems in equidistant spiral motion.In this paper, the differential equation of the spiral is derived through geometric relations, and the evolution laws of position and velocity during the initial coiling process of the spiral are studied. Combined with the rigid body fixed-axis rotation model and the constant velocity condition at the starting point of equidistant spiral motion, the fourth-order Runge-Kutta method and nonlinear equations are used to solve, so as to accurately calculate the position and velocity data of each physical motion in multi-body equidistant spiral motion. At the same time, focusing on the collision of each equidistant spiral motion, the overlap is determined by coordinate discretization and rotation matrix method, which further reveals the collision constraint mechanism of the multi-body joint motion system in the close cooperative motion.This paper not only provides a scientific basis for optimizing the motion trajectory of traditional folk activities, but also offers important theoretical references and methodological inspirations for fields such as motion planning, collision avoidance and cooperative control of complex joint multi-body systems, and has broad application prospects.

Keywords: Equidistant helix, Fourth-order Runge-Kutta method, System of nonlinear equations, Dynamic modeling of multibody systems

1. Introduction

The coordinated control of complex multi-body joint motion systems is of great significance to high-precision assembly in mechanical engineering, collaborative control of robot clusters, multi-component linkage in aerospace, and engineering design. As a typical form of complex multi-body joint motion, multi-body equidistant spiral motion has its trajectory subject to the conservation of spatial helicity and the collaborative constraints of multi-body moving objects. The accurate prediction of its motion trajectory can realize the coordinated control of complex multi-body joint motions. Traditional prediction methods are based on geometric modeling, such as the equidistant spiral geometric modeling method, which has defects such as low prediction accuracy and limited practical application scenarios [1].Based on this, taking the "bench dragon"[2] movement as an example, this paper innovatively establishes a motion model of multi-body equidistant spiral motion using the fourth-order Runge-Kutta method and nonlinear equations, predicts its motion trajectory, and provides a theoretical basis and practical reference for realizing the coordinated control of robot formation[3], group motion planning and multi-body dynamic control.

2. The basic funament of the Fourth-Order Runge-Kutta Method

2.1. The Principle of the Fourth-Order Runge-Kutta Method

The fourth-order Runge-Kutta method is a single-step numerical method for solving initial-value problems of ordinary differential equations. It enables the recurrence of the solution at the next moment y_{n+1} using only the solution at the previous moment yn in each step. Its fourth-order accuracy stems from coefficient matching in the Taylor expansion.

For ordinary differential equations $y'=f(t,y)y(x_0)=y_t$,The iterative formula of the fourth-order Runge-Kutta method is expressed as:

$$y_{n+1}=y_n+\frac{h}{6}(k_1+2k_2+2k_3+k_4) \tag{1}$$

In the formula, the derivatives at the four stages are defined as:

$$\begin{gathered} k_1=f(t_n,y_n) \\ k_2=f\left(t_n+\frac{h}{2},y_n+\frac{h}{2}k_1\right) \\ k_3=f\left(t_n+\frac{h}{2},y_n+\frac{h}{2}k_2\right) \\ k_4=f(t_n+h,y_n+hk_3) \end{gathered} \tag{2}$$

In the formula, $h=X_{n-1}-X_n$denotes the step size, xnrepresents the independent variable at the n-th step, and yn is the corresponding dependent variable[4].

2.2. The Principle of Coordinate Discretization Method for Rigid Body Structures Based on Cartesian Coordinates

2.2.1. The Definition of Coordinate Discretization for Rigid Body Structures:

In the Cartesian coordinate system, coordinate discretization refers to the quantitative description of continuous rigid body motion and spatial attitude through a series of discrete coordinate parameters, thereby converting a complex continuous motion system into a discrete parameter system solvable by mathematical equations. To reduce the parameter dimension and simplify the rigid body coordinate discretization, only the position of the reference point and two unit vectors are retained, $q_i=(\tau_i^T\ u_i^T\ v_i^T)$,The third vector w_i^T can be derived from the two unit vectors through orthogonalization, which still enables the complete description of the rigid body attitude.

2.2.2. The Kinematical Correlation of Discrete Coordinates

In the local coordinate system of a rigid body, the position of point P relative to Oi can be expressed as a linear combination of the local coordinate constants (a,b,c) and the local basis u_i,v_i,w_i,That is, the global expression of rp is:$r_p=r_{Oi}+a_{ui}+v_i+w_i$ In matrix form:

$$r_p=c_i^p q_i \tag{3}$$

$$C_i^n=[l_3 \quad a_3 \quad b_3 \quad c_3]$$

Where I3 denotes the 3*3 identity matrix.

A vector s in the local coordinate system of the rigid body can be expressed as a linear combination of the local basis:$s=d_{ui}+e_{vi}+f_{wi}$, (where (d, e, f) are constants determined by the direction of s)on the rigid body). In matrix form:

$$s=C_i^s q_i \tag{4}$$

$$C_i^*=[0_3 \quad d_3 \quad e_3 \quad f_3]$$

where 03 is a 3*3 zero matrix.

The position and velocity of any point P or vector S on the rigid body can be represented by the linear transformation of discrete generalized coordinates and constant transformation matrices. The discrete coordinates relate the continuous spatial motion through fixed mathematical relationships, achieving the transformation from continuity to discreteness[5].

2.2.3. The Principle of the Rotation Matrix Method

The rotation matrix method is a mathematical approach that transforms a point in the original coordinate system to a new rotated coordinate system, preserving the distance between two points while only changing their orientation. For a point (x, y) in a plane, if it is rotated by an angle around the θ, its rotated coordinates (x', y') can be calculated using the following rotation matrix:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} [6] \quad (5)$$

where 03 is a 3*3 zero matrix.

2.2.4. The Computational Methods for Nonlinear Equations (Systems) Involving Transcendental Equations in MATLAB

The fsolve function solves nonlinear equations using the least squares method. Its general solution form is:X=fsolve(fun,x0,options),where fun is the nonlinear equation to be solved, x0 is the initial value of the variable, and options is a structure generated by the optimset function, which is used to set optimization parameters and can be omitted (using default values)[7].

3. Results

3.1. The Model Parameters and Motion Constraint Settings

To address the establishment of this model, this article will assume a special case for ease of solution and analysis. Assuming that the system is composed of 223 rigid bodies, with the first section being 286cm long and the remaining sections being 165cm long, all rigid bodies have a width of 30cm, and the front and rear sections are connected by handles 27.5cm away from both ends of the rigid bodies. The system will move clockwise along an equidistant spiral with a pitch of 55cm, with each handle located on the spiral. This article will model and analyze the key and difficult points of solving this type of problem within 300 seconds starting from point A in Figure 1.

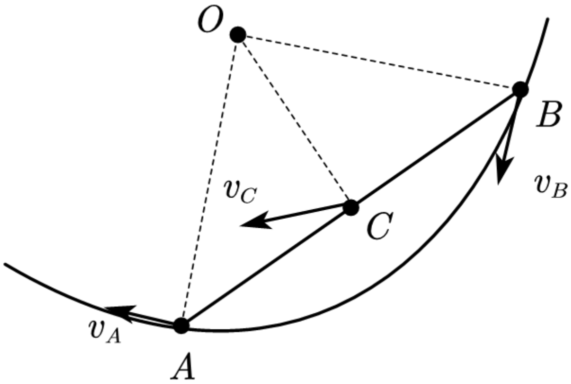

Figure 1. Fixed axis rotation model

3.2. The Derivation of Equidistant Spiral Motion Equations

For equidistant spiral motion, since the radial increase at the same angle is a constant, it can be transformed into polar coordinates for solution, and the following equation can be obtained:

$$x = r\cos\theta \quad y = r\sin\theta \quad (6)$$

And differential equation

$$\frac{dr}{d\theta} = b \quad (7)$$

Solving the differential equation yields,

$$r=a+b\theta \tag{8}$$

Assuming there are two points (r1,θ1) (r2,θ2), if the following relationship exists

$$\begin{gathered} r1=a+b\theta 1 \\ r2 = a + b\theta 2 = a + b\,(\theta 1 + \Delta\theta) \end{gathered} \tag{9}$$

We can obtain the axial distance Δr,

$$\Delta r=b\Delta\theta \tag{10}$$

while the pitch is equivalent to the axial distance after one cycle Δr,

$$p=b\Delta\theta=2\pi b \tag{11}$$

$$b=\frac{p}{2\pi}$$

By organizing, the equation for equidistant spiral lines can be obtained

$$r=\frac{p}{2\pi}\theta \tag{12}$$

And its parameter equation

$$\begin{gathered} x=\left(a+\frac{p}{2\pi}\theta\right)\cos\theta \\ y=\left(\alpha+\frac{p}{2\pi}\theta\right)\sin\theta \end{gathered} \tag{13}$$

3.3. The Kinematic Relationship Modeling of Multi-Rigid-Body System

In order to solve the problem of single collision in the motion of multi system objects, this paper will simulate the dynamic relationships contained in it through a simple model. As shown in the figure, this article considers this process as a single individual AB moving around O on a fixed axis. Given that points A, B, and C all rotate at the same angular velocity ω, the relationship can be derived.

$$\begin{gathered} V_A=V_B=V_C \\ OA=OB=OC \end{gathered} \tag{14}$$

Based on this, the velocity relationship of the individual's head, tail, and middle parts can be obtained

$$\begin{gathered} v_A=\frac{OA}{OC}v_C \\ v_C=\frac{OC}{OB}v_B \\ v_B=\frac{OB}{OA}v_A \end{gathered} \tag{15}$$

The coordinate transformation relationship between Cartesian coordinate system and polar coordinate system [8]

$$\begin{gathered} x=b\theta\cos\theta \\ y=b\theta\sin\theta \\ b=\frac{p}{2\pi} \end{gathered} \tag{16}$$

The tangent vector at the head and tail of the individual can be obtained, that is, the direction of the velocity

$$x=b\cos\theta-b\theta\sin\theta \tag{17}$$
$$y'=b\sin\theta+b\theta\cos\theta$$
$$n_{2i}=(x'_{2i},y'_{2i})$$

According to the vertical relationship of vectors

$$n_i \cdot n_o=0 \tag{18}$$

We can obtain the normal vector ,

$$n=(y'_{2i}-x'_{2i})I \tag{19}$$

the equation of the straight line perpendicular to the velocity direction at the tail (at point B in the figure) can be obtained

$$y'_{2i}(x-x_{2i})-x'_{2i}(y-y_{2i})=0 \tag{20}$$

By combining the linear equations perpendicular to the velocity directions of points A and B, the coordinates of point O on the axis of rotation can be obtained

$$m_i=\frac{y_{2i}-y_{2i-1}}{\frac{x_{2i-1}}{y_{2i-1}}-\frac{x_{2i}}{y_{2i}}}\frac{x_{2i}}{x_{2i-1}}$$
$$n_i=\frac{x_{2i}-x_{2i-1}-\frac{y_{2i-1}}{x_{2i-1}}y'_{2i-1}+\frac{y_{2i}}{x_{2i}}y'_{2i}}{\frac{y_{2i}}{x_{2i}}-\frac{y_{2i-1}}{x_{2i-1}}} \tag{21}$$

From this, the lengths d of OA, OB, and OC can be obtained, and the iterative relationship between the velocities of each point can be calculated based on the model of fixed axis rotation.

$$\frac{ds}{dt}=\mathrm{v}=1 \tag{22}$$

3.4. The Solution Methods for Motion Parameters

Assuming that the head of the first individual in the multi-body motion system has a constant velocity of 1m*s-1, This derivation relationship is shown in Formula 9.

For a segment of arc length microelements in polar coordinates, we can represent it as

$$ds=\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta=\frac{p}{2\pi}\sqrt{\theta^2+1}\,d\theta \tag{23}$$

By combining equations (5) and (6) and integrating them, the following equation can be obtained

$$\frac{p}{2\pi}\int_0^{32\pi}\sqrt{\theta^2+1}\,d\theta=\int_1^0 d\,t \tag{24}$$

It can be observed that both sides of the equation are finite integrals, and a new differential equation can be obtained by taking the derivative

$$\dot{\theta}=-\frac{2\pi}{p\sqrt{\theta^2+1}}\text{\tag11} \tag{25}$$

This equation can be solved using the fourth-order Runge Kutta method to obtain the numerical solution of the function of head position and time[9]. With the position information of the head, the position information of the head and tail of the remaining individuals can be calculated iteratively. The angle corresponding to the position of the handle in the demand solution is θ_{2i}, and the position of the handle that has been solved is (x_{2i-1}, y_{2i-1})

Using di to represent the distance of the i-th handle, the Pythagorean theorem can be used to obtain the following equation

$$\left(\frac{p}{2\pi}\theta_{2i}\cos\theta_{2i}-x_{2i-1}\right)^2+\left(\frac{p}{2\pi}\theta_{2i}\sin\theta_{2i}-y_{i-1}\right)^2=d_i^2 \tag{26}$$

Furthermore, we can obtain

$$f(\theta_{2i})=\left(\frac{p}{2\pi}\theta_{2i}\cos\theta_{2i}-x_{z_{i-1}}\right)^2+\left(\frac{p}{2\pi}\theta_{2i}\sin\theta_{2i}-y_{i-1}\right)^2-d_i^2 \tag{27}$$

In order to determine the position corresponding to the next handle, the value of this function f should be as close to 0 as possible. We use the fsolve function in MATLAB to solve nonlinear equations [10]. At the same time, use a while loop to ensure that the new angle is greater than the current angle θ , and in addition, the obtained new handle should be on the same spiral line as the previous handle.

3.5. The Collision Detection Model and Judgment Logic

In order to ensure continuity during the movement, we should also calculate the time when different individuals collide to provide early warning.

In order to determine the occurrence of a collision, this article calculates the intersection point of the perpendicular lines between two individuals and uses this point as the origin of a new coordinate system. The coordinates of one individual are discretized and rotated to the coordinate system of the second rectangle using a rotation matrix. By determining whether the discrete points fall within the second rectangle, it is determined whether they overlap.

For the i-th individual, the analytical expression for the individual's position is obtained through coordinate relationships

$$(y_{2i}-y_{2i-1})x-(x_{2i}-x_{2i-1})y-x_{2i-1}(y_{2i}-y_{2i-1})+y_{2i-1}(x_{2i-1}-x_{2i})=0 \tag{28}$$

The diagram shows the movement of the individual in Figure 2

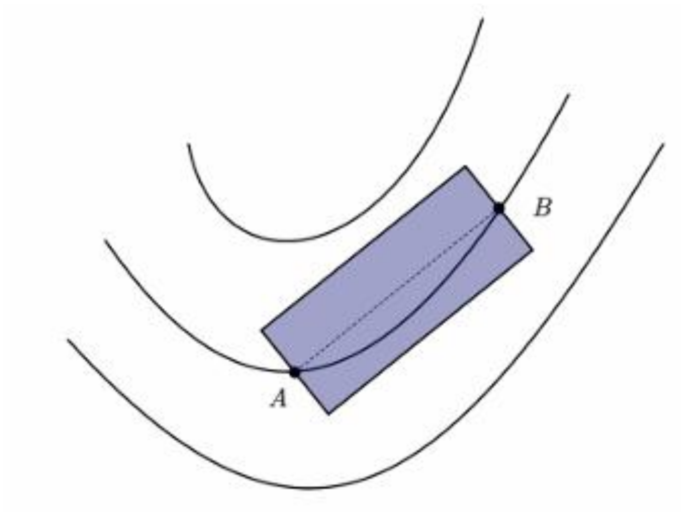

Figure 2. Schematic Diagram of Rigid Body Motion

The equation for the straight line with two borders is solved as follows:

$$C_1=C^+(x_{2i}-x_{2i-1})^2+\left(y_{2i}-y_{2i-1}\right)^2 C_2=C^-(x_{2i}-x_{2i-1})^2+\left(y_{2i}-y_{2i-1}\right)^2 \tag{29}$$

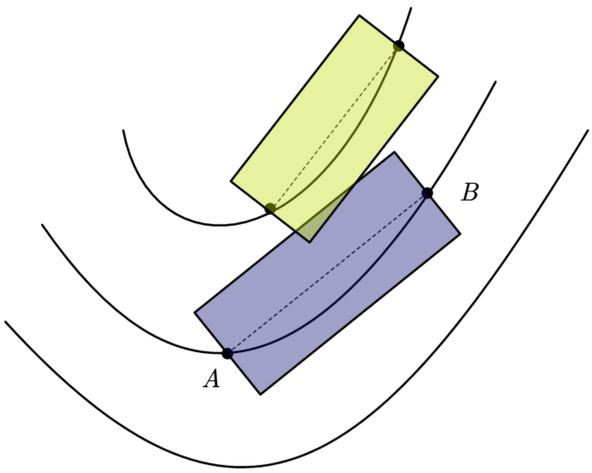

Figure 3. Schematic Diagram of Collision

As shown in the Figure 3, in order to prevent collisions between individuals on two tracks, the influence of their width on this problem must be considered, and the equation can be obtained As shown in the figure, in order to prevent collisions between individuals on two tracks, the influence of their width on this problem must be considered, and the equation can be obtained

$$\frac{|Ax+By+C_1|}{\sqrt{A^2+B^2}}+\frac{|Ax+By+C_2|}{\sqrt{A^2+B^2}}=0.3 \tag{30}$$

Through simulation analysis,we found that the following individuals will definitely pass through the path of the previous individual, so collisions cannot occur in the following individuals, but should occur during the movement of the first individual. Therefore, we only need to focus on the first individual and the one closest to it.

3.6. Simulation Results and Data Presentation

Through modeling and simulation calculations, we found that a collision will occur at 412.6 seconds. The coordinate position and speed of the marked individual during the collision, as well as the coordinate positions of all individuals within 300 seconds, are shown in the Figure 4,5,6 and table 1,2.

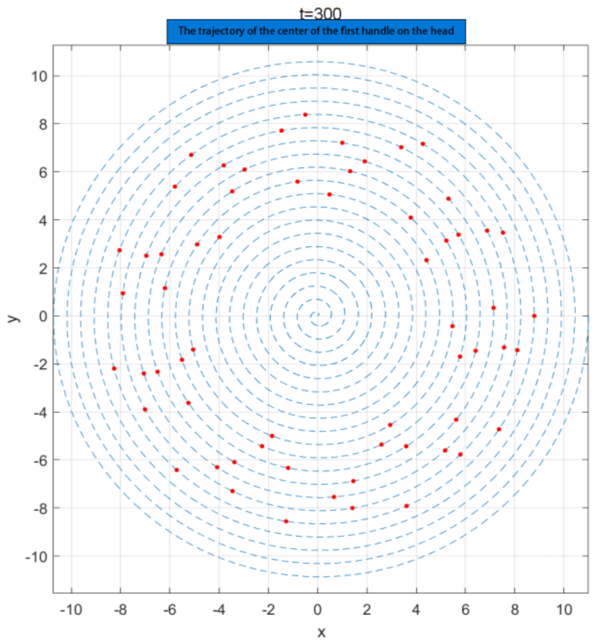

Figure 4. Position of the first handle of the head

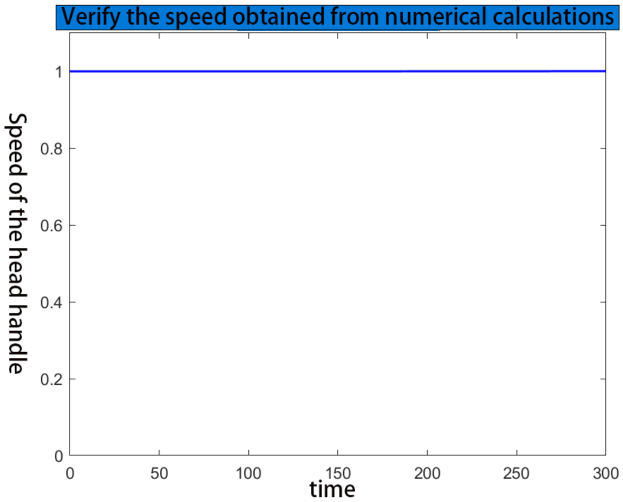

Figure 5. Verification calculation values

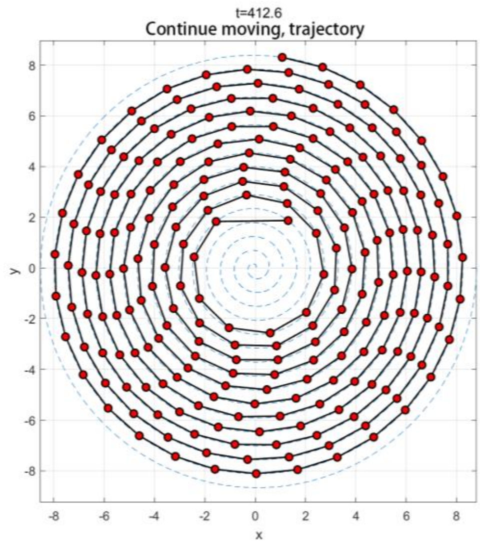

Figure 6. Spiral Entry

Table 1. 0~300s position information of each point

	0s	60s	120s	180s	240s	300s
The first rigid body x (m)	8.8	5.799209	-0.04885	-2.96361	2.594492	4.420274
The first rigid body y (m)	0	-5.77109	-6.30448	6.094781	-5.35674	2.320429
The second rigid body x (m)	8.363824	7.456758	-1.44547	-5.23712	4.82122	2.459488
The second rigid body y (m)	2.826544	-3.4404	-7.40588	4.359628	-3.56195	4.402476
The 52nd rigid body x (m)	-9.51873	-8.68632	-5.54315	2.890456	5.98001	-6.30135
The 52nd rigid body y (m)	1.341137	2.540108	6.377946	7.249289	-3.82776	0.465829
The 102nd rigid body x (m)	2.913983	5.687115	5.361938	1.898794	-4.91737	-6.23772
The 102nd rigid body y (m)	-9.91831	-8.00138	-7.55764	-8.47161	-6.37987	8.936007
The 152nd rigid body x (m)	10.86173	6.682312	2.388757	1.005155	2.96538	7.04074
The 152nd rigid body y (m)	1.828753	8.134544	9.772411	9.424751	8.39972	4.393014
The 202nd rigid body x (m)	4.555102	-6.61966	-10.6272	-9.28772	-7.45715	-7.45866
The 202nd rigid body y (m)	10.72512	9.02557	1.359848	-4.24667	-6.18073	-5.26335
The last rigid body x (m)	-5.30544	7.364557	10.97435	7.838896	3.240153	1.785032
The last rigid body y (m)	-10.6766	-8.79799	0.843473	7.49237	9.469336	9.301126

Table 1. Properties of rigid bodies x,y the coordinate changes cyclically with time, which fully conforms to the geometric characteristics of "equidistant spiral track changes uniformly with increasing angle and radial direction". The coordinate changes of different rigid bodies have cooperative differences, which verifies the multi-body coupling design of "rigid body fixed axis rotation + handle spiral constraint" in the model.

Table 2. Position information of each point when collision

	abscissa x (m)	y-coordinate y (m)	speed (m/s)
The first rigid body	1.607860	1.600338	0.997888
The 2nd rigid body	-1.216456	2.050716	0.989396
The 52nd rigid body	1.761035	4.143629	0.975835
The 102nd rigid body	-1.042984	-5.803967	0.973732
The 152nd rigid body	0.458904	-7.003208	0.972875
The 202nd rigid body	-7.950194	-0.722214	0.972410
The last rigid body	3.045285	7.780078	0.972272

Collision of rigid bodies in Table 2 x,y the coordinates do not overlap with each other, which accurately quantifies the spatial position of the multi-body at the moment of collision. It is proved that the collision detection method of "rotation of discrete coordinates + rectangle overlapping judgment" is effective, and velocity of all rigid bodies is close to each other. 1m/s, same as section 3.4 of the paper "Setting the first rigid body head constant speed" 1m/s" it is proved that the model can transfer the velocity of multibody system stably. The initial condition of the model is consistent with the "velocity iteration relation based on fixed axis rotation".

4. Conclusion

To address this issue, this paper takes the "bench dragon", a traditional folk activity in the Zhejiang-Fujian region,as a typical case to construct a dynamic mathematical model for multi-body equidistant spiral motion. The core method integrates the fourth-order Runge-Kutta method, nonlinear equations, coordinate discretization, and collision detection technology combined with rotation matrices. Specifically, the fourth-order Runge-Kutta method, relying on its global accuracy, enables the accurate calculation of the position and velocity evolution of 223 rigid bodies in the system over time; the nonlinear equation solving framework effectively handles the transcendental function constraints in multi-body coupling; and the collision detection mechanism based on discrete coordinate transformation and rotation matrices accurately determines the overlap between rigid bodies.

The research results show that the established model successfully achieves centimeter-level accuracy in predicting the trajectory of multi-body equidistant spiral motion, and simultaneously reveals the velocity variation law of each rigid body, which is consistent with the actual motion situation. These findings provide a scientific basis for optimizing the trajectory of multi-body equidistant spiral motion and also verify the feasibility of the proposed method in handling complex multi-body coordinated motion.

In summary, this paper breaks through the limitations of traditional geometric modeling and constructs a dynamic modeling framework integrating numerical methods and nonlinear constraint solving. It provides a theoretical reference and practical paradigm for trajectory prediction and collision avoidance of complex multi-body joint motion systems, and can be applied in fields such as robot formation control, multi-component linkage of spacecraft, and high-precision assembly.

Reference

[1] Amersdorfer M, Meurer T. Equidistant tool path and cartesian trajectory planning for robotic machining of curved freeform surfaces [J]. IEEE Transactions on Automation Science and Engineering, 2022, 19 (4): 3311-3323.

[2] Zhang W, Wan W, Wang W. The evolution and review of the cultural ecology of village sports performances: A study on the “Bench Dragon” in Chongren [J]. Academic Journal of Humanities & Social Sciences, 2024, 7 (8): 18-23.

[3] Author A, Author B, Author C. Real-time trajectory tracking control of a parallel robot with flexible multibody model inversion [J]. Journal Name, 2021, Volume (Issue): page-page.

[4] Zhao Q Y, Yu Z H, Li H L, et al. Prediction and calculation of vacuum pump pumping speed curve based on MATLAB/GUI [J]. Vacuum, 2024, 61 (3): 9-12.

[5] Gonçalves S B, Roupa I, Flores P, et al. Kinematic and inverse dynamic analysis using mixed and fully Cartesian coordinates with a generic rigid body [J]. Mechanism and Machine Theory, 2025, 214: 106080.

[6] Zhang Ying, Huang Qineng, Liao Qizheng, et al. Double matrix representation method for the spatial rigid body transformation [J]. Journal of Mechanical Engineering, 2022, 58 (13): 89-100.

[7] Liu M Y, Dong Y Z, Ding H D, et al. Solving methods of nonlinear equations based on MATLAB [J]. Technology Wind, 2017 (14): 256-259.

[8] Kaewkorn H, Zhou L, Li W. RP-Net: A robust polar transformation network for rotation-invariant face detection [J]. Pattern Recognition, 2025, 158: 111044.

[9] Agrawal T, Anand U. Optimization of a Runge–Kutta 4th order method-based airbrake control system for high-speed vehicles using neural networks [EB/OL]. arXiv:2307.12038, 2023.

[10] MathWorks. fsolve function [EB/OL]. Optimization Toolbox, MATLAB R2023a, 2023 [2025/7/16].