Study on the Production Decision by Dynamic Programming and Nonlinear Programming

Yuxin Cheng * Yongchen Lu, Qi Qian
Soochow University Jiangsu, China
* Corresponding Author Email: freesia255@163.com

Abstract. An excellent production decision-making plan is the fundamental guarantee for a company to operate continuously and generate profits. This paper aims to establish a decision-making model by using csampling inspection, dynamic optimization and nonlinear programming, and provide the optimal solution through boundary conditions and constraints, so as to realize the optimization of production decision-making. This paper first establishes a hypothesis testing model, using different testing methods for various sample sizes to determine the minimum sample size at a given confidence level. Then, through simulation experiments, the sampling strategy is optimized, and the defect rate and confidence interval are provided. Subsequently, by combining dynamic programming and nonlinear 0-1 programming, and considering the mutual influence between processes, the production flow of multi-stage processes and spare parts is optimized, resulting in the optimal production procedure. The findings of this study provide effective references for decision optimization in complex production conditions, offering significant practical application value.

Keywords: Production decision-making, binomial distribution, hypothesis testing, dynamic optimization, nonlinear 0-1 programming.

1. Introduction

In electronics markets, product quality directly impacts a company's competitiveness. Production decision-making involves resource allocation, production planning, method selection, quality control, cost management, and risk assessment to optimize predefined objectives. High-confidence sampling and production step adjustments based on qualified rates can significantly reduce production losses, boosting profitability. Therefore, effective sampling inspection and process optimization models are crucial.

Sequential probability ratio testing (SPRT) is a method for hypothesis testing that compares the likelihood ratio of two hypotheses after each data point, helping to reduce tests and minimize costs[1]. Studies by Tang Guoping et al. [2] and Zhao Pan et al. [3] explored SPRT for exponential and Poisson distributions, while Tian Zixuan et al. [4] applied sequential testing to improve computational efficiency in distributed testing. Despite these advancements, most studies remain theoretical and lack real-world production integration.

The Cost of Quality (COQ) concept, introduced by Juran in 1951, divides quality costs into prevention, appraisal, internal failure, and external failure costs [5]. Freiesleben et al[6]. improved COQ by incorporating technological advances and quality improvements. In 2012, Zhang Lianying et al.[7] further optimized the progress, cost, and quality model using an immune-genetic particle swarm algorithm. Additionally, the introduction of intelligent algorithms has provided high-precision and high-efficiency solutions to complex problems[8]. These studies provide the theoretical foundation and computational models for quality cost control, but the models are relatively complex and primarily used for theoretical analysis, making them difficult to apply in actual production settings.

This paper first employs a sequential testing model to optimize the sampling inspection method and determine the minimum sample size. Using dynamic optimization and nonlinear 0-1 programming methods, the traditional quality cost model is improved by decomposing the entire production process into related sub-processes, optimizing decision variables and constraints to enhance the model's accuracy and applicability. This approach holds the potential to provide reliable decision-making solutions for actual production processes.

2. Sampling Inspection Model

This model aims to determine the minimum number of sampling inspections under a given confidence level. The key to selecting a sampling inspection plan for parts lies in determining the sample distribution type based on sample size, that is, whether it exceeds the nominal value at a given confidence level. Therefore, this paper adopts a one-sided testing method. Samples have only two states: pass and fail. For small samples, the distribution follows a binomial distribution. For large samples, according to the Central Limit Theorem, when np>10[9], the binomial distribution can be approximated by a normal distribution as shown in Fig 1.

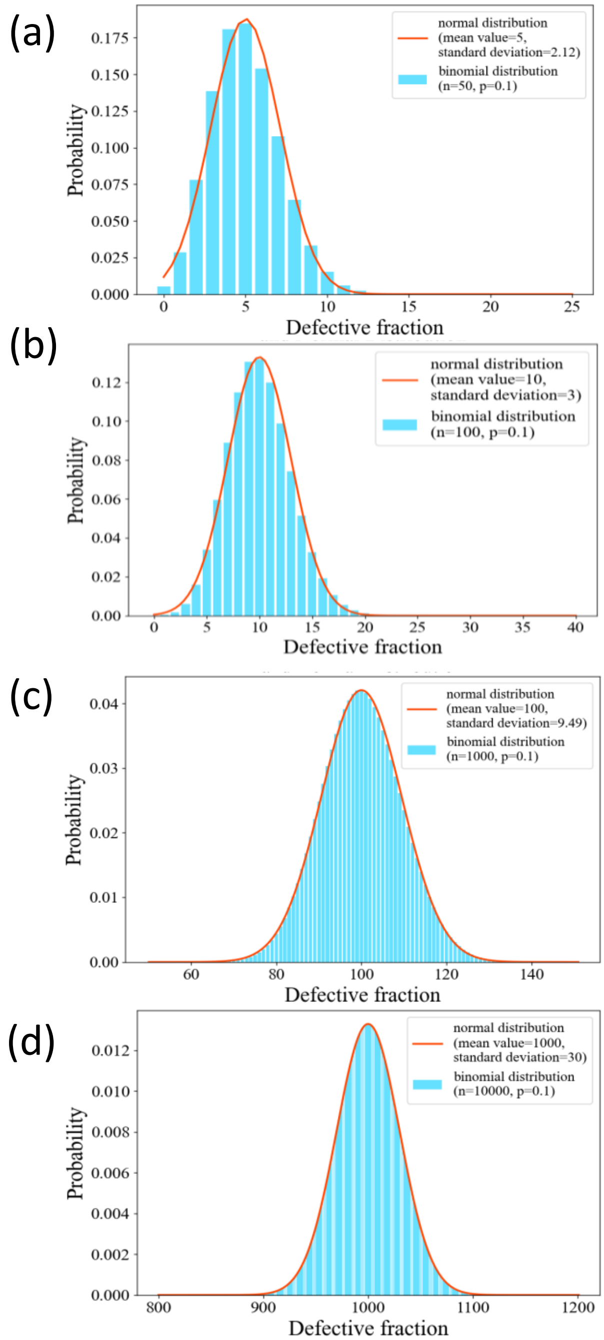

Fig. 1 Binomial distribution under sample sizes of 50 (Fig. 1 a), 100 (Fig. 1 b), 1000 (Fig. 1 C), 10000 (Fig. 1 d).

2.1. Sample Size and Model Construction

First, based on the conditions for approximating a binomial distribution to a normal distribution: when np>10 and for a defect rate of 10%, samples with fewer than 100 parts are defined as small samples, and samples with more than 100 parts are defined as large samples.

For sample sizes of 50, 100, 1000, and 10,000, graphical fitting was performed (Fig. 1). It can be observed that when the sample size is 50 (i.e., under the small sample condition), the distribution of the binomial distribution differs significantly from the normal distribution. When the sample size is 100 (i.e., at the critical condition), the distribution of the binomial distribution shows some resemblance to the normal distribution. It can be inferred that as the sample size tends to infinity, the binomial distribution becomes indistinguishable from the normal distribution. Therefore, this paper divides the sampling inspection process into two cases: small samples and large samples.

(1) Hypothesis Testing for Small Samples Using Binomial Distribution

For small samples, this paper adopts a one-sided sequential binomial distribution test to quickly determine whether to accept or reject a product. A strong binomial test is used in this study to determine with high probability whether the null hypothesis satisfies the specified conditions[10]. This approach overcomes the disadvantage of significance testing, where accepting the null hypothesis only means failing to reject it, and improves the precision of the test. he one-sided confidence upper limit p_U^+ and lower limit p_L^+ for the binomial distribution parameter p at a confidence level γ are given by the following two formulas[11]:

$$\sum_{k=0}^{d} C_n^k \left(p_U^+\right)^k \left(1-p_U^+\right)^{n-k} = 1-\gamma \tag{1}$$

$$\sum_{k=0}^{d-1} C_n^k \left(p_L^+\right)^k \left(1-p_L^+\right)^{n-k} = \gamma \tag{2}$$

From these formulas, the upper and lower confidence distribution functions for the binomial distribution parameters $F_U(p)$ and $F_L(p)$ are respectively:

$$F_U(p) = \sum_{k=d+1}^{n} C_n^k p^k (1-p)^{n-k} \tag{3}$$

$$F_L(p) = \sum_{k=d}^{n} C_n^k p^k (1-p)^{n-k} \tag{4}$$

From equations (3) and (4), the upper and lower limits for the sequential test under the strong test can be determined.The test includes the following three steps (Fig. 2): a) Sample extraction: A sample is randomly drawn from the total population, typically consisting of two samples. b) Set the limit: set the parameter n and the number of defective products D in the binomial distribution according to the selected sample, and determine the upper confidence distribution function $F_U(p)$ and the lower confidence distribution function $F_L(p)$ from the formula in the figure, that is, the upper and lower limits of the strong test. c) Hypothetical judgment:

If $F_U(p_0) \ge \gamma$, then $p < p_0$ can be determined at the confidence level γ;

If $F_U(p_0) \le \gamma$, then $p > p_0$ can be determined at the confidence level γ;

If $F_U(p_0) < \gamma$ and $F_L(p_0) > 1-\gamma$, the amount of sample information is too small to make a judgment, so it is necessary to increase the n value of the sample, which is usually increased step by step.

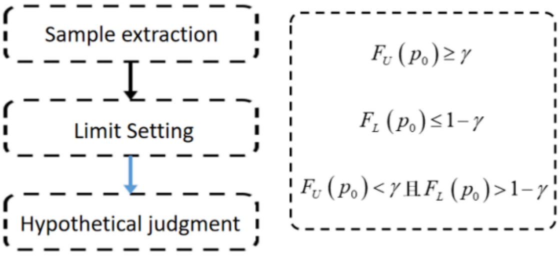

Fig. 2. Flow chart of inspection steps.

(2) Hypothesis Testing for Large Samples Using Normal Distribution

For large sample sizes, the binomial distribution can be approximated by a normal distribution. The test checks whether the distribution probability is greater than or less than the given nominal value P_0. This paper adopts a one-sided normal distribution hypothesis test. The specific testing steps are as follows:

Set the Hypotheses: assume H_0 is that the defective rate p does not exceed P_0, The alternative hypothesis H_1 is that the defect rate p of the spare parts exceeds p_0, as:

$$H_0 : p \leq p_0, H_1 : p > p_0 \tag{5}$$

a) Given the significance level: According to the confidence requirements, the significance level is specified, which refers to the probability of making an error when the population parameter falls within a certain interval.

b) Determine the Z-score(Table I): To determine the number of samples required at different significance levels, it is necessary to find the critical values, or the Z-scores, corresponding to different confidence levels in the normal distribution. The relationship between the confidence level and the significance level is:

$$\gamma = 1 - \alpha \tag{6}$$

Table 1. Z-scores at Different Confidence Levels

Confidence level	90%	95%	99%
Z-score	1.280	1.645	2.330

c) Determine the sample size: According to the basic principles of statistical hypothesis testing, using normal approximation, the sample size ***n*** can be calculated using the following formula:

$$n = \frac{Z_\alpha^2 p_0 (1\text{-}p_0)}{d^2} \tag{7}$$

where Z is the Z-score corresponding to the different confidence levels, p_0 is the nominal value, and d is the allowable error.

d) Decision rule: The decision rule, based on the minimum sample size ***n*** calculated from Equation (7), is given by the following formula:

$$X > np_0 \tag{8}$$

Where X is the number of defective items in the sample, n is the minimum sample size, and p_0 is the nominal value. If $X > np_0$, there is sufficient reason to reject the null hypothesis; otherwise, the null hypothesis is accepted.

2.2. Specific example application

(1) 95% Confidence Hypothesis Test

In the case of small samples, a strong test using the binomial distribution sequential test is employed. Here, the paper assumes a sample size of 100. Under $p_0 = 0.1$ and confidence level $\gamma = 0.95$, the paper uses Python to conduct 50,000 Bernoulli experiments. For each experiment, a strong test is performed on sample sizes ranging from 2 to 100, and the number of tests required to complete the strong test is recorded. This allows determination of the minimum sample size needed to reject the null hypothesis. The results are shown in Fig. 3.

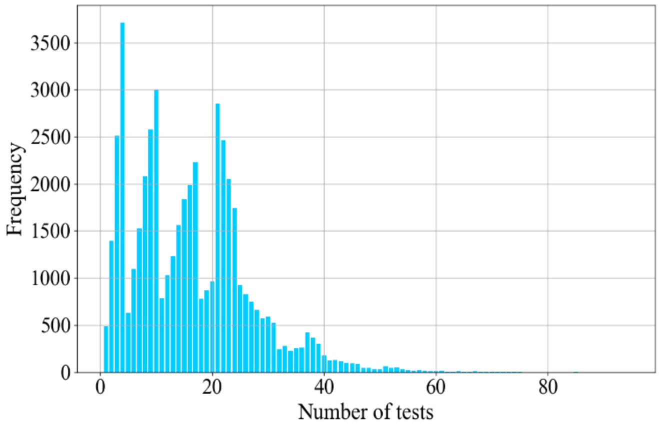

Fig. 3. The distribution chart of detection times for strict inspection under 95% confidence level.

In the 50,000 Bernoulli experiments with a sample size of 100, the most frequent number of tests required to conclude the strong test (i.e., to determine whether to reject the null hypothesis) was 28. The frequency of tests sharply decreased when the number of tests exceeded 33. Considering the randomness of sampling, when the number of tests is small, the risk of the experiment results is high, and the confidence is low. When the number of tests exceeds 33, the frequency of test counts required to conclude the strong test significantly decreases. Therefore, when the number of parts is 100, the company can obtain more accurate results after 33 samples are tested.

In the case of a large sample, where the total number of parts exceeds 100, a normal distribution hypothesis test is used. With the same parameters and an allowable error of 3%, the minimum sample size n=271 is calculated. According to the decision rule: based on Equation (8), if the number of defective items in the 271 samples exceeds 28, the null hypothesis is rejected, meaning the defect rate is considered to be greater than 10%. Otherwise, the null hypothesis is accepted, meaning the defect rate is considered to be less than 10%.

(2) 90% Confidence Hypothesis Test

Similarly, in the case of small samples, the strong test using the binomial distribution sequential test is chosen. In this paper, a sample size of 100 is also selected, with the parameters set as follows: $p_0 = 0.1$ and $\gamma = 0.90$. Python was used to conduct 50,000 Bernoulli experiments, with each experiment performing the strong test on sample sizes ranging from 2 to 100. The results are shown in Fig. 4.

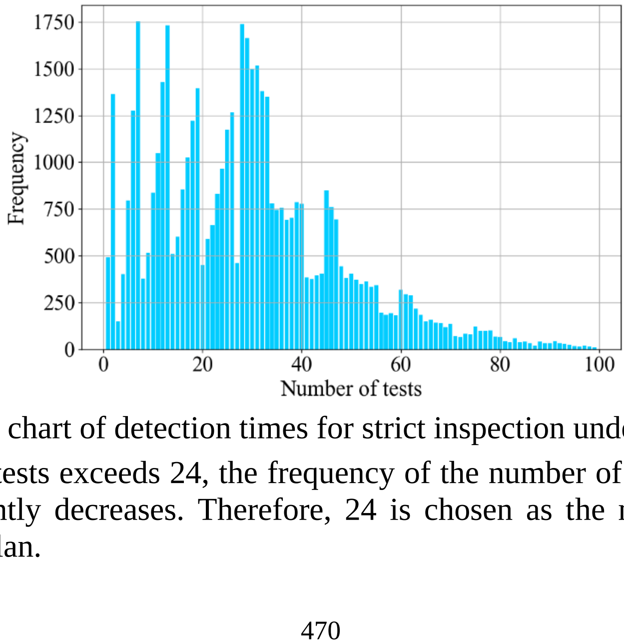

Fig. 4. The distribution chart of detection times for strict inspection under 90% confidence level.

When the number of tests exceeds 24, the frequency of the number of tests required to conclude the strong test significantly decreases. Therefore, 24 is chosen as the minimum number of tests required for the testing plan.

In the case of a large sample, with the same parameters and an allowable error of 3%, the minimum sample size n=164 is determined. According to the decision rule, if the number of defective items in the 164 samples exceeds 17, the null hypothesis is rejected, meaning the defect rate is considered to be greater than 10%. Otherwise, the null hypothesis is accepted, meaning the defect rate is considered to be less than 10%.

(3) Model Optimization

In the minimum sample sizes given above, there exists a detection gap, meaning that when the number of parts exceeds 100, the minimum sample size required for the normal distribution test is greater than the total number of parts. In this case, it is necessary to optimize the existing strategy. Below are two specific methods:

a) Use of Binomial Distribution Sequential Test for Strong Test in the Gap Area: Although when the number of parts exceeds 100, the normal distribution can statistically approximate the binomial distribution, the fit in certain numerical intervals is not high. In this case, using a binomial distribution sequential test for the strong test can help fill this gap, sacrificing the number of tests to improve the accuracy of detection.

b) Increase the Allowable Error: According to Equation (7), in normal distribution hypothesis testing, reducing the minimum sample size can be achieved by increasing the allowable error d, lowering the accuracy to reduce the minimum number of tests. Fig. 5 presents the minimum sample sizes under different allowable errors.

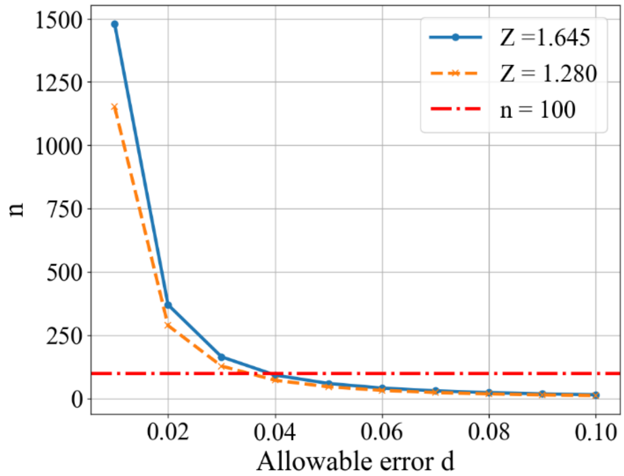

Fig. 5. The relatio nship between minimum sample size n and allowable

As shown in Fig. 5, when the allowable error exceeds 4%, there is no sampling gap between the binomial distribution and normal distribution tests.

3. Multi Process Production Decision Model

In the production process of a company, there are often m stages and n components involved. Factors such as the defect rate of each component, raw materials, assembly and test losses, and market sales need to be considered. The goal is to provide a specific production decision plan to maximize revenue.

This paper constructs a model that divides the production and sales process into different stages, with the objective of minimize losses. The model calculates the dynamically changing defect rates and recovery values at each stage, and provides constraint conditions and boundary adjustments for model solving.

3.1. Stage Division

This paper simply divides the production process of n raw components and m assembly processes into four stages:

Component Stage: The first stage involves n raw components, each with a defect rate. These components either undergo testing or are directly assembled.

Semi-finished Product Stage: From the second to the m-1 stage, multiple components (or semi-finished products) are combined to form new semi-finished products. These semi-finished products may be further selected for testing and assembly.

Finished Product Stage: The m-th stage involves assembling multiple semi-finished products into finished products. These finished products can either undergo testing or proceed directly to the market.

Disassembly Stage: This stage includes disassembling finished products and semi-finished products that fail internal inspections, as well as disassembling products returned by customers. Components can be disassembled into semi-finished products or completely broken down into raw components.

3.2. Model Construction

Based on the above production process, this paper provides the objective function for minimizing production loss, which is the sum of "component parameter " "semi-finished product parameter," "finished product parameter," "disassembly parameter," "compensation parameter," and "defective parameter," minus the "recovery parameter."

When multiple components or semi-finished products are combined, it is assumed that the precision of each inspection is 100%. The defect rate P_{pre} from the previous step is the combined defect rate of these components or semi-finished products. The defect rate from the previous step is given by:

$$P_{pre} = 1 - \prod_{i=1}^{n_{pre}} (1 - P_i \cdot (1 - x_i)) \tag{9}$$

Where P_i is the original defect rate of the i-th component or semi-finished product; x_i indicates whether the i-th, component or semi-finished product has been inspected, with 1 representing "inspected" and 0 representing "not inspected"; n_{pre} represents the number of components or semi-finished products involved in the combination from the previous step.

Substituting P_{pre} into theconditional probability formula, the following equation is obtained:

$$P'_{pre} = (1 - x_x) \frac{1 - \prod_{i=1}^{n_{pre}} (1 - P_i \cdot (1 - x_i))}{1 - \prod_{i=1}^{n_q} (1 - P_i \cdot (1 - x_i))} \tag{10}$$

Where n_q is the number of components or semi-finished products combined in the current process, and x_x represents the inspection status of the current semi-finished product or finished product.

Inspection loss and assembly loss include the sum of inspection and assembly consumption for components, semi-finished products, and finished products. Defective product handling losses include the consumption of disassembling semi-finished products, internally disassembling finished products, and disassembling finished products returned by customers. The disassembly recovery revenue is dynamically adjusted based on the conditional defect probability and defect rate.

Under the assumption that the inspected products are considered qualified, compensation losses only occur when uninspected defective products enter the market. Under inspection conditions, the loss generated by not choosing to disassemble defective products due to inspection failure and directly discarding the selected ones needs to be derived from previous step losses, such as component loss and inspection loss. This is a dynamic variable.

3.3. Model Solution

(1) Objective Function

The objective function, which represents the minimum production loss, is composed of the following conditions: Objective Function = Component parameter + Semi-finished Product parameter

+ Finished Product parameter + Disassembly parameter + Compensation parameter + Defective parameter - Recovery parameter

(2) State Transition Equation

The optimal value function $V_i(s_i)$ for stage i represents the minimum loss starting from stage i.The recursive equation and state transition equation can be expressed as:

$$V_i\left(s_i\right)=\min\left\{C_i\left(s_i,a_i\right)+V_{i+1}(s_{i+1})\right\} \tag{11}$$

$$s_{i+1}=f(s_i,a_i) \tag{12}$$

Where $C_i\ (s_i,\ a_i)$ includes the detection, assembly, static disassembly losses, and compensation losses at the current stage.

(3) Determine Conditions

At the final finished product stage, the boundary condition is:

$$V_m(s_m)=C_m(s_m) \tag{13}$$

This represents the net loss from inspection, compensation, and disassembly at the final stage.

The constraints consist of the following parts:

inspection selection for components, semi-finished products, and finished products:

$$x_i\in\{0,1\},\ y_i\in\{0,1\},\ z_i\in\{0,1\} \tag{14}$$

Each decision variable must be either 0 or 1, indicating whether inspection is selected.

Disassembly and inspection constraints: Semi-finished products and finished products can only be disassembled if they are found to be defective after inspection. Therefore, it is necessary to ensure the constraint between disassembly and inspection:

$$d_j\le y_j,d_f\le z \tag{15}$$

Disassembly operations can only be selected if the semi-finished products or finished products have been inspected and found to be defective.

Inspection after disassembly constraint: If the disassembled products are to be inspected, it must be ensured that they have undergone the disassembly operation beforehand.

$$p_j\le d_j,\ p_f\le d_f \tag{16}$$

The choice of inspection after disassembly depends on whether disassembly has been performed.

3.4. Example Calculation and Analysis

Taking the case of two processes and eight components as an example, the specific relationships between components, semi-finished products, and finished products are as follows: Semi-finished product 1 is composed of components 1, 2, and 3; semi-finished product 2 is composed of components 4, 5, and 6; semi-finished product 3 is composed of components 7 and 8. The finished product is made from semi-finished products 1, 2, and 3. Specific parameters are shown in Table II.

Table 2. Parameters for Components, Semi-finished Products and Finished Products

Component	Defect Rate	Purchase Unit Price	Inspection Cost	Semi-finished Product	Defect Rate	Assembly Cost	Inspection Cost	Disassembly Cost
1	10%	2	1	1	10%	8	4	6
2	10%	8	1	2	10%	8	4	6
3	10%	12	2	3	10%	8	4	6
4	10%	2	1	Finished Product				
5	10%	8	1		10%	8	6	10
6	10%	12	2					
7	10%	8	1		Selling Price		Replacement Loss	
8	10%	12	2		200		40	

According to the objective function, constraints, and boundary conditions, the data from the above table is substituted into the model, and the solution is obtained using software programming. The result is that when the production loss is minimized, no inspection is performed on components 1 to 8 and semi-products 1 to 3, while the finished product is inspected. If the finished product or semi-products are found to be defective, they are not disassembled and reused. The maximum revenue obtained is 67.6.

4. Conclusion

This paper addresses the sampling inspection and process optimization in enterprise production, proposing a decision-making model based on dynamic optimization and nonlinear programming. The model uses binomial distribution sequential testing and confidence testing to ensure sample size accuracy and efficiency. By combining dynamic programming and nonlinear 0-1 programming, it is closely linked with the actual production of the enterprise. The model's accuracy can be improved by adding reference factors and constraints, and integrating intelligent algorithms like decision trees or Markov chains can enhance decision precision and flexibility, providing stronger decision-making support for enterprises.

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